TRR 154

Mathematical modeling, simulation, and optimization using the example of gas networks

Subproject C03

Admissible Robust Nodal Control

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Abstract

Our goal is the development of a theoretical basis for nodal control problems with hyperbolic PDEs and stochastic boundary data. Nodal control means the single control elements are distributed in the network.
Hereby, an adequate framework for the system dynamics based on the actual stochastic is to be developed, risk neutral and risk averse objective functions – motivated by applications – are to be considered, the existence of optimal controls is to be proven and necessary optimality conditions are to be derived. The analysis of the optimal value function is not only required in the treatment of stochastic, but also interesting from the viewpoint of parametric optimization.

Model

Isothermal Euler-Equations

$$\rho_t + q_x = 0$$

$$q_t + p_x + \left(\frac{q^2}{\rho}\right)_x = -\frac{\lambda}{2D}\frac{q|q|}{\rho} - g\rho s_{\text{lope}}$$

p	pressure	$\frac{\lambda}{D}$	friction coefficient
q	mass flow	s_{lope}	slope
ρ	density	g	gravitational constant

State Equation

$$p = RTz(p)\cdot\rho$$

$$z(p) = 1 + \alpha p$$

α	slope in the linear model for the compressibility factor
R	specific gas constant
T	temperature

Subsonic (e.g. $\frac{q}{\rho} = |v| \leq c$) stationary solutions for $s_{\text{lope}} = 0$:

q constant

$$p(x) = F^{-1}\left(F(p_0) - \tfrac{1}{2}q|q|\int_{x_0}^{x}\frac{\lambda(s)}{D}ds\right)$$

$$F(p) := \frac{1}{\alpha}p + \left(q^2RT - \frac{1}{\alpha^2}\right)\ln(|1+\alpha p|) - q^2RT\ln(p)$$

Initial Situation

- Optimization on gas networks almost exclusively for the stationary case.
  Simulation on gas networks has been done for dynamic models.
- Stochastic optimization well developed for the finite-dimensional (mixed-integer) linear case.
  Only singular results for the nonlinear infinite-dimensional case.
- Combination of stochastic optimization with boundary control of PDEs is pioneering.
- Existence of regular solutions on arbitrary networks unclear.

Challenges

- In general, regular solutions of hyperbolic systems can break down after finite time.
- Uncertain boundary data and its interactions with the decisions have to be considered in the optimal control problem.
- Existence of regular system states and optimal controls is investigated on graphs.

Work schedule

WT 1: Model choice

- Stochastic: Nonanticipativity as new constraint, risk neutral or risk averse measure
- PDE: Hyperbolic PDEs with stochastic boundary data

WT 2: Well-posedness of the system dynamics

- Semilinear hyperbolic relaxation

WT 3 : Regularity of solutions

- Existence of regular solutions for hyperbolic quasilinear PDEs

WT 4: Optimal control for stochastic boundary data

- Existence of optimal controls

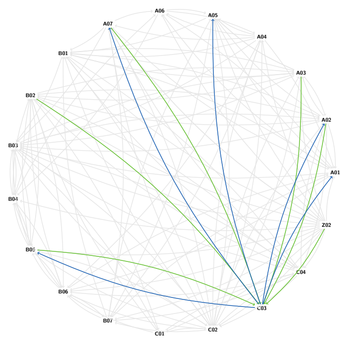
WT 5: Optimality conditions

- Stochastic nature of the objective function, adjoint equation, convergence of optimality conditions

WT 6 : Sensitivity analysis

- Lipschitz-continuity and one-sided differentiability of the optimal value function

Position in the TRR154



A02: Comparison of relaxation concepts (parabolic-hyperbolic)
A03: Discussion of hyperbolic relaxation
A07: Comparison of Models: Discrete – PDE
B02: Confidence intervals for friction coefficients
B05: Stochastic Optimization
Z02: Data

A01: Nodal control instances
A02: Comparison of relaxation concepts (parabolic-hyperbolic)
A05: Nodal control instances
A07: Comparison of Models: Discrete – PDE
B05: Stochastic Optimization with PDE