Subproject B06

Robustification of Physical Parameters in Gas Networks

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TRR 154
Mathematical modeling, simulation, and optimization using the example of gas networks

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Challenges in Gas Networks under Uncertainty

gas network with active elements and flows q, pressure p

$$\sum_{a\in\delta^+(v)} q_a - \sum_{a\in\delta^-(v)} q_a = q_a^{\text{nom}} \qquad \forall v \in V$$

$$p_u^2 - p_v^2 = \phi_a q_a |q_a| \qquad \forall a = (u,v) \in A$$

active elements

$$p_v \in [\underline{p}_v, \overline{p}_v] \qquad \forall v \in V$$

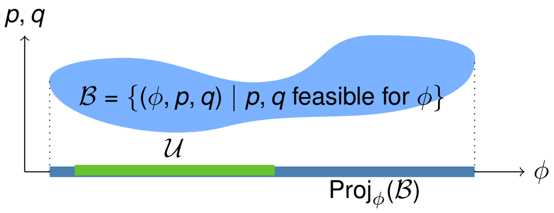
possible sources of uncertainty:
demand $q^{\text{nom}} \in \mathcal{D}$, *roughness* $\phi \in \mathcal{U}$

goal:
is there a configuration of the active elements such that there is a feasible pressure/flow for every value in the uncertainty set?

challenge:
multi-stage non-convex mixed-integer robust optimization problem

Passive Networks: Set Containment Approach

- ▷ robust feasible: $\forall \phi \in \mathcal{U}\ \exists$ feasible $p, q \iff \mathcal{U} \subseteq \text{Proj}_\phi(\mathcal{B})$
- ▷ leads to polynomial optimization problems (SDP approximation hierarchy)



Deciding Feasibility

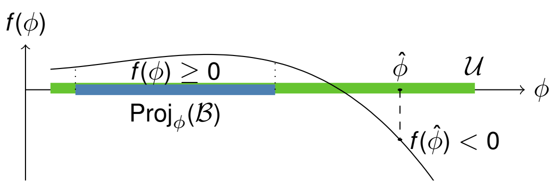
- ▷ **result:** $\mathcal{U} \subseteq \text{Proj}_\phi(\mathcal{B}) \iff$
  $\mathcal{G} = \{x \mid g_j(x) = 0,\ j \in J\} \subseteq \mathcal{H} = \{x \mid h_i(x) \leq 0,\ i \in I\}$,
  for g_j, h_i polynomial functions
- ▷ need to check if for all $i \in I$: $\max_{x\in\mathcal{G}} h_i(x) \leq 0$

computational experiments for feasible one-cycle networks:

nodes	#probs	hierarchy level 2	hierarchy level 3	hierarchy level 4	mean cpu [s] 2	mean cpu [s] 3	mean cpu [s] 4	#infeas.
3	6	6	0	0	0.5	3.5	6.1	0
4	12	5	1	6	0.9	3.7	30.0	6
5	20	10	0	10	1.0	8.7	198.0	0

Deciding Infeasibility

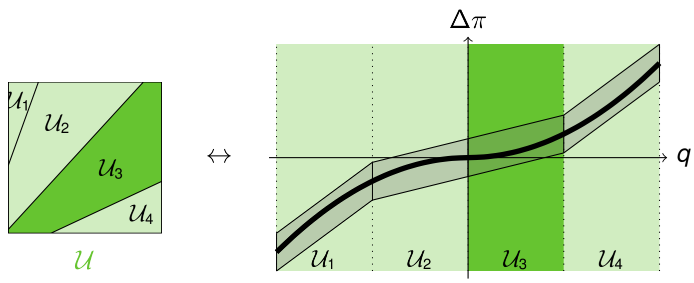
- ▷ find polynomial f which is non-negative on $\text{Proj}_\phi(\mathcal{B})$ but is negative for some $\hat{\phi} \in \mathcal{U}$
- ▷ **result:** f exists if $\mathcal{U} \setminus \text{Proj}_\phi(\mathcal{B})$ contains an open subset



Preprint: D. Aßmann, D. den Hertog, F. Liers, M. Stingl, J. Vera. *Deciding Robust Feasibility and Infeasibility Using a Set Containment Approach: An Application to Stationary Passive Gas Network Operations*

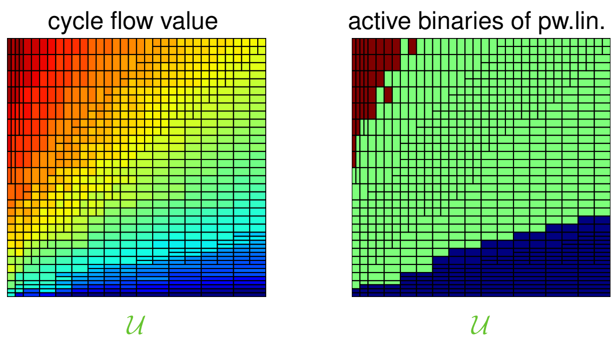
Active Networks: Splitting the Uncertainty Set

- ▷ piecewise linear relaxation of pressure-drop functions
- ▷ so far: works on networks with edge-disjoint cycles
- ▷ **result:** segments of linearization map to partitions of $\mathcal{U}$
- ▷ splitting of $\mathcal{U}$ allows elimination of auxiliary binary variables for piecewise linearization on second stage



Experimental Splitting in Strict Robust Model

- ▷ three-node network with two uncertain pipes
- ▷ adaptive splitting of $\mathcal{U}$ until feasibility is reached
- ▷ 6 iterations (732 partitions)



Contributions

- ▷ Demo 3: validation of solutions via set containment, maximization of additional gas flow via splitting approach
- ▷ Uncertainty Team (→ talk by René Henrion)

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