TRR 154

Mathematical modeling, simulation, and optimization using the example of gas networks

Stochastic Optimization in Gas Transport

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Polynomial: $f_0(r, z)$ = 792309173492573/10000000000000000 * z0 * *2 +(790293177289401/10000000000000000) * z0 * z1 +(19790769160733/1250000000000000) * z0 * z2
+(1977611827901 41/250000000000000) * r * z0 −(21795594683461/1250000000000000) * z0 +(3951465886447/100000000000000) * z1 * *2 +(790293177289401/10000000000000000) * z1 * z2
+(474129624942157/1000000000000000) * r * z1 −(118543976593 41/250000000000000) * z1 +(791630766429319/1000000000000000) * z2 * *2 +(949864200187469/1000000000000000) * r * z2
−(949956919715183/1000000000000000) * z2 +(300821531786803/1000000000000000) * r * *2 −(513873813856 97/1250000000000) * r +(13636648067908 1/2000000000000000) * 1;

Key Techniques

Computation of the feasibility probability of exit loads in stationary gas networks:

▷ Reduced NLP

▷ Spheric-Radial Decomposition (B04)

▷ Solving systems of polynomial equations using Gröbner basis approach

Feasibility Probability

Pressure loss vector from root node to each node:

$$g : \mathbb{R}^{|V|-1} \times \mathbb{R}^{|N|} \to \mathbb{R}^{|V|-1},$$
$$g(u,v) := (A_B^\top)^{-1}\Phi_B|A_B^{-1}(u - A_N v)|(A_B^{-1}(u - A_N v))$$
$$g_0(u,v) := 0 \in \mathbb{R}$$

Set of feasible load vectors M consists of all b such that there exists q_N with:

$$A_N^\top g(b, q_N) = \Phi_N|q_N|q_N$$
$$\min_{i \in V}\left[(p_i^{max})^2 + g_i(b,q_N)\right] \geq \max_{i\in V}\left[(p_i^{min})^2 + g_i(b,q_N)\right]$$

$$\mathbb{P}\left\{b \in \Omega \mid b(\omega) \in M\right\}$$

Reduced NLP

A^+	Graph $G = (V, E)$ with incidence matrix
b^+	Balanced in- and outlet
Φ	Matrix of friction coefficients for pipes

Kirchhoff I and II for flow q and pressure p^+:

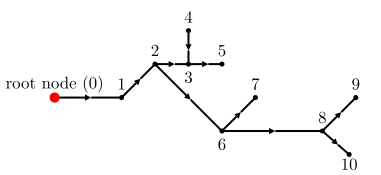
$$A^+ q = b^+ \quad \text{and} \quad -A^{+\top} p^{+2} = \Phi|q|q.$$

v_0	Root node
$A = (A_B, A_N)$	After deletion of the first row

$q = (q_B, q_N)$, $p^+ = (p_0, p)$, $b^+ = (-\mathbb{1}^\top b, b)$

Resulting explicit flow and pressure profiles:

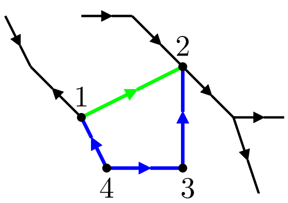
$$p^2 = p_0^2 \cdot \mathbb{1} - (A_B^\top)^{-1}\Phi_B|A_B^{-1}b - A_B^{-1}A_N q_N|(A_B^{-1}b - A_B^{-1}A_N q_N)$$



$$p_6^2 = p_0^2 - \phi_{01}q_{01}|q_{01}| - \phi_{12}q_{12}|q_{12}| - \phi_{26}q_{26}|q_{26}|$$

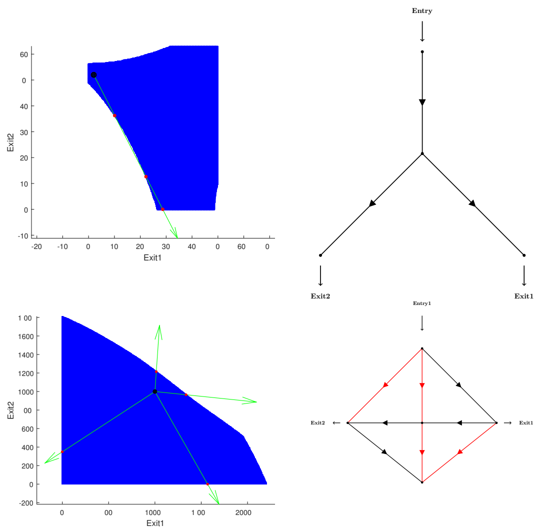
$$0 = \mathcal{F}(q_N) := A_N^\top(A_B^\top)^{-1}\Phi_B|A_B^{-1}b - A_B^{-1}A_N q_N|(A_B^{-1}b - A_B^{-1}A_N q_N) - \Phi_N|q_N|q_N.$$

Operator $\mathcal{F} : \mathbb{R}^{|N|} \to \mathbb{R}^{|N|}$ is continuous, monotone, and coercive.



$$\phi_{12}q_{12}|q_{12}| = -\phi_{14}q_{14}|q_{14}| + \phi_{43}q_{43}|q_{43}| + \phi_{32}q_{32}|q_{32}|$$

Examples



Blue	Feasible region M
Green	Samples (spheric-radial decomposition)
Red	Points of intersection (Gröbner basis)

... [Gröbner Basis for a network with 3 cycles]