Subproject B02

Parameter id., sensor localization and quantification of uncertainties in switched PDE-systems

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TRR 154

Mathematical modeling, simulation, and optimization using the example of gas networks

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The project concentrates on inverse problems related to the transport of gas in networks. The main focus lies on parameter identification for the underlying physical process, like quantifying the friction coefficient, or leakage detection within the network. Moreover, robustification of the identification process and efficient numerical methods for these problems are subject of investigation.
Within the CRR 154, the project participates in demonstrator D1.

Identification of the Friction Coefficient

Output Least Squares Problem

$$\min_{\lambda \in L^2(x_L, x_R)} \frac{1}{2}\|y^d(t) - \rho(t, x_R)\|^2_{L^2(0,T)}$$

for the transport process described by a semilinear model

$$\begin{aligned} \rho_t + q_x &= 0 \\ q_t + a^2 \rho_x &= -\lambda \frac{q|q|}{\rho} \end{aligned} \quad \text{on } (0,T) \times (x_L, x_R) \qquad \text{(ISO 2)}$$

with initial and boundary conditions

$$q(0,\cdot) = q_0(\cdot),\ \rho(0,\cdot) = \rho_0(\cdot) \qquad \text{(IC)}$$

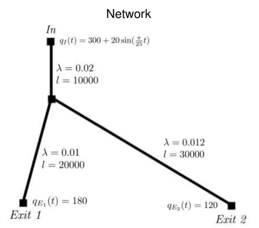
$$q(\cdot, x_L) = q_L(\cdot),\ q(\cdot, x_R) = q_R(\cdot) \qquad \text{(BC)}$$

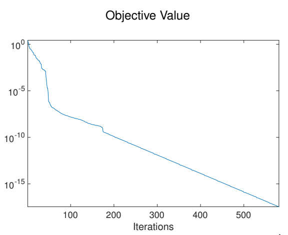
Results

- ▷ Existence of broad solutions for the underlying problem.
- ▷ Sensitivity properties of the solution operator for (??).
- ▷ Extension of results to passive networks.
- ▷ Existence of solutions for the identification problem if $\|\lambda\|_{\mathcal{X}}$ bounded with compact embedding $\mathcal{X} \hookrightarrow L^2(x_L, x_R)$.

Numerical Validation

- ▷ Demonstrator 1 test-network, $T = 200$.
- ▷ Pipe-wise constant friction coefficient ($\lambda_{Ini} \equiv 0.01$).
- ▷ Varying volume flow-supply, constant consumption.
- ▷ Particle management discretization method with $\Delta x = 10$.
- ▷ Steepest Descent method for NLP.





Original coefficients approximated with $\|\lambda^* - \lambda\|_{l^2} < 4 \cdot 10^{-11}$.

Future Challenges:

- ▷ Deriving stationarity system for λ-identification.
- ▷ Leakage detection.
- ▷ Quantification of uncertainties for parameter.

Optimal Control of Nonlinear Scalar Conservation Laws

Model Problem: Identification of Initial Condition

$$\min_{u \in L^\infty(\mathbb{R})} \int_{\mathbb{R}} G(y(T,\cdot)) + R(u)$$

$$\text{s.t.} \quad y_t + [f(y)]_x = 0 \text{ in } (0,T) \times \mathbb{R} \qquad (1)$$

$$y(0) = u \text{ in } \mathbb{R}$$

with regularization term $R(\cdot)$ and strictly convex flux $f \in C^2(\mathbb{R})$.

Adjoint-Consistent Discretization for (??) with RK Methods

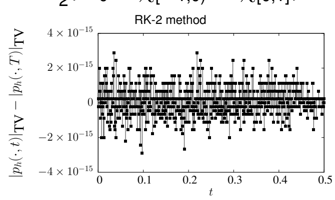
Method of lines and suitable numerical flux f^Δ provide

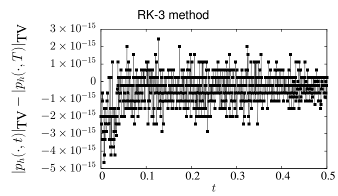
$$\dot{y}_i = \Delta x^{-1}(f^\Delta(y_{i-1}, y_i) - f^\Delta(y_i, y_{i+1})),\ y_i(0) = u_i \qquad (2)$$

- ▷ Applying any TVD-RK method to (??) provides a TVD-RK discrete adjoint but with conjugated coefficients.
- ▷ Imposing strong stability preserving (SSP) property to both, discrete state and adjoint, results in a first order method.
- ▷ If the TVD-RK method is SSP and at most of order three, the discretization of the adjoint is *consistent* with (??).
- ▷ Imposing SSP to the discrete state is sufficient to obtain a TV-*stable* discrete adjoint.

TV-stability of Adjoint Discretization

$G(y(T,\cdot)) = y(T,\cdot)^5 - y(T,\cdot)$, Burgers equation ($f(y) = \frac{1}{2}y^2$), $T = \frac{1}{2}$, $u_0 = \chi_{[-1,0)} - \chi_{[0,1]}$, f^Δ - Engquist-Osher





Cooperations within the TRR 154

- ▷ *Parameter identification in a semilinear hyperbolic system*
  Preprint with H. Egger, T. Kugler *TU Darmstadt*
- ▷ Mixed-Integer Optimal Boundary Control of Semilinear Transport Systems
  Partner: F. Hante, G. Leugering *FAU Erlangen-Nürnberg*
- ▷ Adjoint-Consistent High Resolution Discretization Methods for Scalar Balance Laws
  Partner: S. Ulbrich *TU Darmstadt*

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