Subproject A07

Efficient Network Flow Models for Instationary Gas Flows

TRR 154

Mathematical modeling, simulation, and optimization using the example of gas networks

Martin Groß Marc Pfetsch Martin Skutella

Technische Universität Berlin

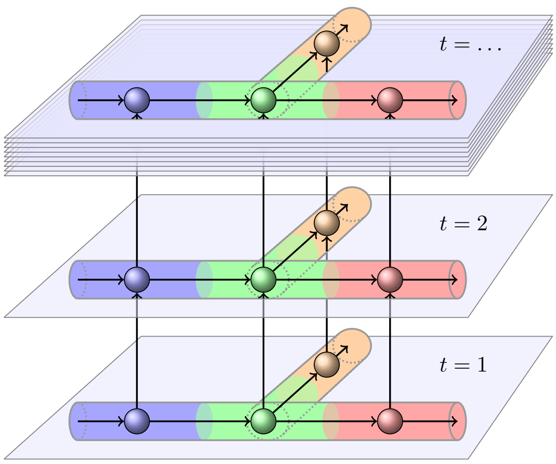
www.tu-berlin.de

Summary

The goal of this subproject is to study the applicability of network flow theory to simplified models of instationary gas networks and related transport networks. Important issues are the complexity of the studied time-expanded models, especially with respect to integral decisions and complex network elements, as well as the development of approximation algorithms and a characterization of infeasibility, especially in the instationary case.

Time-expanded Network Flow Model

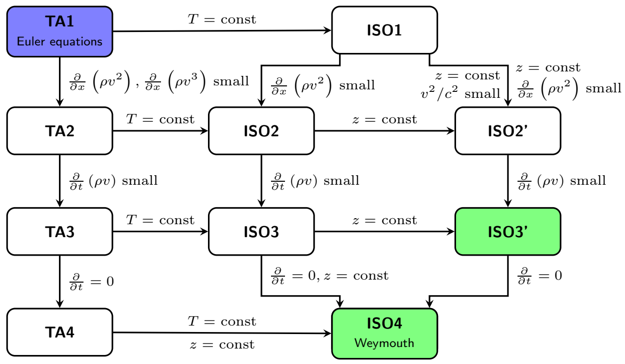
Space and time discretization of the pipeline network leads to a time-expanded network where each node represents a segment of the network in a time interval. Nodes that are adjacent in space or time are connected by edges.



A07 in the Model Hierachy

Assumptions:

- ▷ Constant temperature, compressibility factor and speed of sound.
- ▷ Friction depends only on relative roughness.
- ▷ v^2/c^2 and $\frac{\partial}{\partial t}(\rho v)$ very small.



The Potential-Induced Flow Problem

Nodes have pressures $p(v,t)$ and edges e have mass flows $q(e,t)$ at all times t. The most important constraints are of the following types:

Flow Conservation:

$$\underbrace{q(v_t, v_{t+1})}_{\text{gas to next time point}} - \underbrace{q(v_{t-1}, v_t)}_{\text{gas from last time point}} + \overbrace{\sum_{e\in\delta^+_G(v)} q(e,t)}^{\text{flow from outgoing arcs}} - \underbrace{\sum_{e\in\delta^-_G(v)} q(e,t)}_{\text{flow from incoming arcs}} = \overbrace{b_{v,t}}^{\text{injection/withdrawal}}$$

Flow-Pressure Relation for an edge $e = (v,w)$:

$$\underbrace{\beta_e}_{\text{constant}} \underbrace{q(e,t)|q(e,t)|}_{\text{gas flow on } e \text{ (squared)}} = \underbrace{p(v,t)^2}_{\text{pressure in } v \text{ squared}} - \underbrace{p(w,t)^2}_{\text{pressure in } w \text{ squared}}$$

They are linked by constraints of the form $p(v,t) = \gamma_v q(v_{t-1}, v_t)$. In addition, we have non-negativity constraints for pressures and mass flows to the next time layers.

Techniques

Stepwise Forward Computations:

- ▷ Easy, but requires that the initial state of the network is known.

Convex Minimum Flows:

- ▷ The quadratic constraints can be "pulled up" into a cubic objective.
- ▷ Derivative of the cubic objective yields the quadratic constraints.

Network Topology:

- ▷ Exploit the topology of gas-networks. Graph-classes like trees, (generalized) series-parallel graphs, graphs with bounded tree-width, etc. allow for faster algorithms.
- ▷ Gas pipeline networks often belong to such classes, e.g.:
  - ◦ Tree: Greek gas network
  - ◦ Generalized Series-Parallel: GasLib582 (Germany)

Generalized Series-Parallel Decomposition:

- ▷ Uses the inherent structure of generalized series-parallel graphs.
- ▷ Problem can be reduced to solving the problem on an edge, and the three compositions (serial, parallel, tree) of subnetworks.
- ▷ Applications: E.g., determine valve settings to achieve a given mass flow and node pressures.

Computational Complexity:

- ▷ Allows to separate hard and easy settings. E.g., determining the feasibility of a nomination in a network with pipes and valves is
  - ◦ strongly NP-hard in general graphs,
  - ◦ weakly NP-hard in generalized series-parallel graphs,
  - ◦ easy in trees.