TRR 154

Mathematical modeling, simulation, and optimization using the example of gas networks

Subproject A03

Mixed Integer-Continuous Dynamical Systems with Partial Differential Equations

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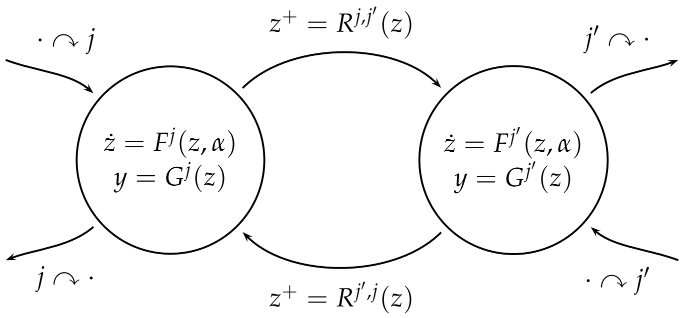
Problem Statement

Gas dynamics on a network is an alluring connection between discrete and continuous mathematics in the sense that the gas flow is driven, on the one hand, by continuous evolution equations on each pipe and, on the other hand, by time-discrete switching processes of valves and compressors. This project's motivation is the control of the gas dynamics by choosing optimal switching decisions for such active elements. To this end we develop a general theory for mixed integer-continuous dynamical systems driven by a set of PDEs that are coupled by switching rules of either an explicit or a state-dependent implicit type. The sought results, however, promise a gain of insights in hybrid systems far beyond the difficulty of gas networks.



Mixed integer-continuous system. The state z and the mode j evolve in time $t \geq 0$ by provision of

- ▷ a PDE generated by F^j (while j is constant),
- ▷ a jump generated by $R^{j,j'}$ (when switching from j to j').

Optional: $\alpha(t)$, $y(t)$ for input- or output-signals.

Initial Situation

Model gas density ϱ and flux q on each pipe of the network by

$$\begin{aligned} \frac{\partial \varrho}{\partial t}(t,x) + \frac{\partial q}{\partial x}(t,x) &= 0, \\ \frac{\partial q}{\partial t}(t,x) + c^2 \frac{\partial \varrho}{\partial x}(t,x) &= -\frac{\lambda}{2D}\frac{q(t,x)|q(t,x)|}{\varrho(t,x)}, \end{aligned} \qquad \text{(ISO2)}$$

and coupling conditions:
(1) Density is continuous at each node.
(2) Flows must sum up to zero at each node.
Active elements are nodes that can be switched:
· valves can be open/closed
· compressors can be on/off
A cost functional penalizes operational costs, the violation of gas demands, etc.

Task: Operate active elements by optimal switching decisions!

Challenges

- ▷ integer-continuous parametrization of time and space with relevant multiscale interpretation of the dynamics
- ▷ well-posedness: continuous dependency on initial data and Zeno phenomena
- ▷ adjoint equation and gradients with respect to variation of switching times and order
- ▷ existence of optima, necessary optimality conditions, optimization methods

Work Schedule

WT 1: systems with semigroups

- ▷ hybrid systems of semilinear PDEs with strongly continuous semigroups and transition maps at switching points
- ▷ existence and regularity of solutions, independent of the switching sequence
- ▷ sensitivity analysis with respect to switching times and order, adjoint calculus and necessary optimality conditions

WT 2: application to gas

- ▷ semigroup formulation for (ISO2) on a network
- ▷ optimal switching of valves and compressors, numerical proof of concept implementation

WT 3: feedback-controlled switching decisions

- ▷ formulation of state-dependend switching rules
- ▷ existence of solutions, continuous dependence on initial data, zeno phenomena
- ▷ convergence of solutions in the sense of graphs on hybrid structures (in time and space)

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