TRR 154

Mathematical modeling, simulation, and optimization using the example of gas networks

Subproject A02

Optimal control of hyperbolic PDE-models with switched controls in gas networks

Stefan Ulbrich   Johann Schmitt

Technische Universität Darmstadt

www.tu-darmstadt.de

Motivation

Hyperbolic balance laws with switching and state constraints arise in the context of various problems, e.g.:

- ▷ Modelling and optimal control of gas transport
  - ∘ switching of network components, e.g. valves
  - ∘ pressure bounds
- ▷ Optimal control of traffic flow
  - ∘ switching of traffic lights
  - ∘ avoidance of traffic jams

On/Off Switching Problem

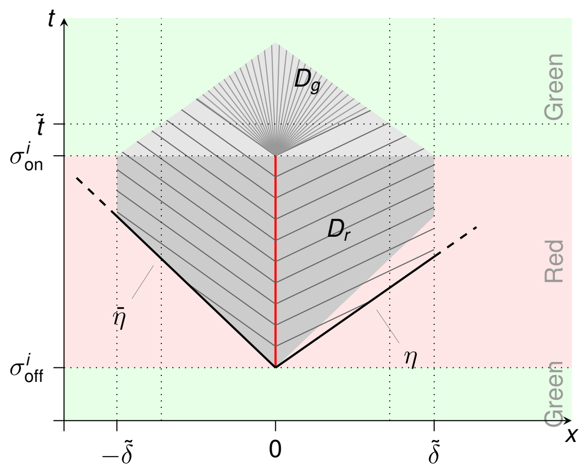
$$\min_{\sigma} J(y(\sigma)) := \int_a^b \psi\left(y(\bar{t}, x, \sigma), y_d(x)\right)\, dx \qquad \text{(P)}$$

under the constraints

$$\begin{aligned}
y_t + f(y)_x &= g(\cdot, y), && \text{on }]\sigma_{on}^i, \sigma_{off}^{i+1}[\times\mathbb{R}\\
(y_1)_t + f(y_1)_x &= g(\cdot, y_1), && \text{on }]\sigma_{off}^i, \sigma_{on}^i[\times\mathbb{R}^-\\
(y_2)_t + f(y_2)_x &= g(\cdot, y_2), && \text{on }]\sigma_{off}^i, \sigma_{on}^i[\times\mathbb{R}^+\\
y(0, \cdot) &= u_I, && \text{on } \mathbb{R},\\
y(\sigma_{on}^i, \cdot)\big|_{\mathbb{R}^-} &= y_1(\sigma_{on}^i -, \cdot), && \text{on } \mathbb{R}^-\\
y(\sigma_{on}^i, \cdot)\big|_{\mathbb{R}^+} &= y_2(\sigma_{on}^i -, \cdot), && \text{on } \mathbb{R}^+\\
y_1(\sigma_{off}^i, \cdot) &= y(\sigma_{off}^i -, \cdot)\big|_{\mathbb{R}^-}, && \text{on } \mathbb{R}^-\\
y_2(\sigma_{off}^i, \cdot) &= y(\sigma_{off}^i -, \cdot)\big|_{\mathbb{R}^+}, && \text{on } \mathbb{R}^+\\
y_1(\cdot, 0-) &= 0, && \text{on }]\sigma_{off}^i, \sigma_{on}^i[\\
y_2(\cdot, 0+) &= 1, && \text{on }]\sigma_{off}^i, \sigma_{on}^i[
\end{aligned} \qquad \text{(Z)}$$

$$\sigma \in \Sigma_{ad} \subset [0, T]^{2n_\sigma+2}, \quad y(\bar{t}, \cdot) \leq \bar{y}(\cdot), \quad \text{on } [a, b], \qquad \text{(S)}$$

where $f \in C^2_{loc}(\mathbb{R})$ has the property $f(0) = f(1) = 0$ and is uniformly convex.



Moreau-Yosida regularization

Use Moreau-Yosida regularization to treat the state constraints in (S):

$$\min_{\sigma} J_\gamma(y(\sigma)) := J(y(\sigma)) + \frac{1}{2\gamma}\int_a^b \left(y(\bar{t}, x, \sigma) - \bar{y}(x)\right)_+^2 dx \qquad (P_\gamma)$$

under the constraints $\quad y(\sigma)$ solves (Z), $\quad \sigma \in \Sigma_{ad}$

Results

Results for the on/off switching problem:

- ▷ Differentiability of the reduced cost functional w.r.t. σ
- ▷ Adjoint-representation of the objective function gradient
- ▷ Existence of optimal solutions for (P)
- ▷ Necessary optimality conditions for (P)
- ▷ Convergence analysis of $(P_\gamma)_{\gamma>0}$ for $\gamma \to 0$:
  - ∘ Convergence of optimal solutions of $(P_\gamma)_{\gamma>0}$
  - ∘ Convergence of Lagrange multiplier estimates

Future Work

Extension of the results for the scalar on/off switching problem to the case of systems (Isothermal Euler equations):

$$\begin{cases}
\partial_t \rho_i + \partial_x(\rho_i v_i) = 0, \quad i \in E,\\
\partial_t(\rho_i v_i) + \partial_x(\rho_i v_i^2 + p_i(\rho_i)) + g\rho_i \partial_x h_i\\
\qquad + \dfrac{\lambda_i}{2D_i}\rho_i v_i |v_i| = 0, \quad i \in E,\\
K(\rho(t, 0), v(t, 0), \rho(t, L), v(t, L), u(t)) = 0,\\
\rho(0, x) = \rho_0, \quad v(0, x) = v_0,
\end{cases} \qquad \text{(ISO1)}$$

- ▷ First consider the case of $|E| = 1$ and piecewise smooth initial data (Generalized Riemann Problem):

$$(\rho_0, v_0)(x) := \begin{cases} (\rho_1, v_1)(x) & \text{if } x < 0\\ (\rho_2, v_2)(x) & \text{if } x > 0 \end{cases}$$

- ▷ Results for semilinear Systems (currently in progress):
  - ∘ Shift-differentiability of the state w.r.t. the control
  - ∘ Fréchet-differentiability of the reduced cost functional

References

[1] S. Pfaff and S. Ulbrich. Optimal Boundary Control of Nonlinear Hyperbolic Conservation Laws with Switched Boundary Data. *SIAM Journal on Control and Optimization*, Vol. 53 (2015), No. 3, pp. 1250–1277.

[2] S. Pfaff and S. Ulbrich. Optimal Control of Nonlinear Hyperbolic Conservation Laws by On/Off-Switching. To appear in *Optimization Methods and Software, 2016.*

[3] J. M. Schmitt and S. Ulbrich. Optimal Control of Nonlinear Hyperbolic Conservation Laws by On/Off-Switching with State Constraints. (In preparation)

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