Logic and Computability SS24, Assignment 5

Due: 22. 05. 2024, 23:59

1 Satisfiability Modulo Theories

1. [2 points] Given the formula

$$f(x) = g(x) \vee z = f(y) \rightarrow f(z) \neq g(y) \wedge x = z.$$

Apply the Ackermann reduction to compute an equisatisfiable formula in $\mathcal{T}_E$.

2. [2 points] Given the formula

$$\varphi_{EUF} := f(a,b) = x \wedge f(x,y) \neq g(a) \vee f(m,n) = b \vee f(g(a),y) \neq a.$$

Apply the Ackermann reduction to compute an equisatisfiable formula in $\mathcal{T}_E$.

3. [2 points] Perform the graph-based reduction to translate the following formula in $\mathcal{T}_E$ into an equisatisfiable formula in propositional logic.

$$\varphi_{EUF} \quad := \quad x \neq y \wedge y = g_x \vee g_x = g_y \rightarrow \neg(g_y \neq z \vee z = f_x) \wedge \neg(f_x = f_y \wedge x \neq z)$$

4. [2 points] Perform the graph-based reduction to translate the following formula in $\mathcal{T}_E$ into an equisatisfiable formula in propositional logic.

$$a \neq b \wedge b = c \vee c = d \rightarrow \neg(d \neq e \vee e = f) \wedge \neg(f = g \wedge a \neq e)$$

5. [2 points] Consider the following formula in the conjunctive fragment of $\mathcal{T}_{EUF}$.

$$\varphi_{EUF} \quad := \quad f(b) = a \wedge e = b \wedge c = f(c) \wedge d \neq f(e) \wedge f(a) = f(d) \wedge a \neq f(c) \wedge d = f(a)$$

Use the congruence closure algorithm to determine whether this formula is satisfiable.

6. [5 points] Use the lazy encoding approach to check whether the formula φ in $\mathcal{T}_{EUF}$ is satisfiable.

$$\begin{aligned}\varphi :=& ((f(a) = b) \lor (f(a) = c) \lor \neg(b = c)) \land ((b = c) \lor (a = b) \lor (f(a) = b)) \land \\ & (\neg(f(a) = b) \lor (a = b)) \land ((b = c) \lor \neg(a = b) \lor \neg(f(a) = b)) \land \\ & (\neg(f(a) = c) \lor (b = c)) \land (\neg(f(a) = c) \lor (b = c) \lor \neg(a = b)) \land \\ & ((f(a) = b) \lor (f(a) = c))\end{aligned}$$