

Logic and Computability SS24, Assignment 4

Due: 15. 05. 2024, 23:59

1 Natural Deduction for Predicate Logic

For each of the following sequents, either provide a natural deduction proof, or a counterexample that proves the sequent invalid.

For proofs, clearly indicate which rule, and what assumptions/premises/intermediate results you are using in each step. Also clearly indicate the scope of any boxes you use.

For counterexamples, give a complete model. Show that the model satisfies the premise(s) of the sequent in question, but does not satisfy the respective conclusion.

1. [2 points] $\forall x (P(x) \wedge Q(x)) \quad \vdash \quad \exists x (P(x) \vee Q(x))$
2. [3 points] $\exists x \neg P(x) \quad \vdash \quad \neg \forall x P(x).$
3. [2 points] $\exists x (P(x) \vee Q(x)) \quad \vdash \quad \exists x P(x) \vee \exists x Q(x)$
4. [2 points] $\exists x \neg P(x), \exists x \neg Q(x) \quad \vdash \quad \exists x (\neg P(x) \wedge \neg Q(x))$
5. [3 points] $\forall x (P(x) \vee Q(x)), \quad \forall x (\neg P(x)) \quad \vdash \quad \forall x (Q(x))$
6. [3 points] $\forall a \forall b (P(a) \wedge Q(b)) \quad \vdash \quad \forall a \exists b (P(a) \vee Q(b))$