Logic and Computability SS24, Assignment 2

Due: 10. 04. 2024, 23:59

1 Natural Deduction for Propositional Logic

For each of the following sequents, either provide a natural deduction proof, or a counterexample that proves the sequent invalid.

For proofs, clearly indicate which rule, and what assumptions/premises/ intermediate results you are using in each step. Also clearly indicate the scope of any boxes you use.

For counterexamples, give a complete model. Show that the model satisfies the premise(s) of the sequent in question, but does not satisfy the respective conclusion.

1. [2 points] $x \wedge (y \wedge z) \vdash (x \wedge y) \wedge z$
2. [3 points] $p \wedge q \vee r \vdash (p \vee r) \wedge (q \vee r)$
3. [3 points]
   (a) $\vdash \neg(\neg p \vee q) \vee p$
   (b) $\vdash \neg p \vee (\neg q \vee p)$
4. [3 points] $(p \to q) \wedge (q \to r), p \vdash \neg\neg r \wedge \neg p$
5. [4 points] $\neg(q \wedge p) \vdash \neg q \vee \neg p$