Non-inertial motion of charged superconducting systems

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Abstract

The possibility of detecting a constant gravitational field and of detecting gravitational radiation using charged superconducting detectors is considered. It is shown that such fields do have effects upon the instruments discussed, and that in principle these effects might be measured.

It is shown however that there are several serious sources of noise in practical instruments. These noise sources would mask the signal by several orders of magnitude in the practical instruments we have considered. It does not appear feasible to reduce the noise sufficiently to be able to measure the effects of the fields.

1 Introduction

Late last century Rowland [1] demonstrated that the charges on a capacitor generate magnetic fields when set into rotation. Theoretical and experimental investigations have recently been performed on this effect using superconducting apparatus. It has been possible to detect angular velocity relative to the local inertial frame using the effect [2,3,4].

Here we investigate theoretically the possibility of using charged superconducting systems to measure non-inertial motion other than rotation. We consider designs of superconducting apparatus which might be able to measure a constant gravitational field, and also versions which might be sensitive to gravitational radiation.

Although it is theoretically possible to build superconducting devices of this sort, there are several, serious sources of noise in practical instruments. These noise sources would mask the signal by several orders of magnitude in the practical instruments we have considered. It does not appear feasible to reduce the noise sufficiently to be able to measure the effects described here.

An analysis of instruments which may be sensitive to gravitational fields is given in section 2, and a superconducting gravity-wave detector is analysed in section 3.

2 Accelerometers

At first sight it appears to be possible to build a superconducting device with no moving parts, which is capable of detecting a gravitational field. See figure 1 . Two weak links A and B are connected together in parallel with a wire of superconductor, and a voltage V is applied across them. The apparatus is placed in a gravitational field $\underline{g}$ as shown. A frequency counter at the top of the apparatus near A can compare the frequencies of the radiations from the two weak links.

Since the weak links are connected together with superconductor, one might expect the differences in electrochemical potential across the two links to be identical. In analysing the consequences assuming that this is the case, we shall arrive at a contradiction and thus demonstrate that this assumption was wrong.

The frequency f of the radiation emitted by a weak link is related to the difference in electrochemical potential V across the link through the equation $h f = 2 e V$, where h is Planck's constant and e is the charge on the electron [5]. Since by assumption the weak links each have the same difference of electrochemical potential V across them, then the frequencies emitted are also identical. The counter near A measures the expected value for the frequency of radiation from A , whereas the radiation from B will be red shifted in travelling through the gravitational field, so that a difference of frequency will be measured, indicating the strength of the gravitational field.

The amount of the red shift can be calculated from energy conservation. A photon of frequency f possesses mass $m = h f / c^2$, so that it loses energy $dE = m g x$ in travelling distance x through the gravitational field g . The change in frequency is therefore given approximately by $df / f = g x / c^2$. If the frequency change were not given by this formula, it would be possible to construct a perpetual motion machine (6,7).

One could imagine constructing a simpler and improved version of the gravity 'detector' as follows. Intense electromagnetic radiation from a single source irradiates both weak links, and the phases of the weak links are locked to the radiation in the usual way. Because of the red shift, the frequency at the upper link is lower and so there is a smaller potential difference across this link. This drives current around the circuit: the current builds up with time according to $L\, dI/dt = V_B - V_A$ so that a noiseless integration is performed internally. A SQUID could be used to detect this current.

The expected performance of such a device is rather impressive. A device $x = 1\text{m}$ long in the earth's gravitational field g irradiated with a frequency $f = 1$ THz would be expected to allow flux to enter the circuit at a rate $df = f\, g\, x / c^2 = 10^{-4}$ flux quanta per second. Such a flux could easily be detected using a SQUID after only a few seconds.

As a matter of fact, such devices have already been built (14,15) in experiments designed to test the material independence of the a.c. Josephson effect. Unfortunately they did not have sufficient sensitivity to measure the gravitational effects. Differing superconducting materials were used for the weak links, but otherwise the apparatus was as described above. A null result was obtained in preliminary experiments to the level of $df/f = 2\ 10^{-9}$.

It would be possible to build a perpetual motion machine if the physics were as described above. Suppose that an apparatus at A allows two electrons to cross the barrier and the energy $E = 2\, e\, V$ is converted at A into a photon which is sent to the bottom of the apparatus. At the bottom B there is a similar machine which works in reverse: it takes the energy from the photon and uses it to drive a pair of electrons up against the potential difference. The energy required to do this would be again $E = 2\, e\, V$. There would therefore be some extra energy available because of the blue shift of the photon, which could be extracted from the perpetual motion machine. Clearly, the assumption that $V_B = V_A$ is not correct; it must be that the potential differences at the top and bottom are related approximately by:

$$(V_B - V_A) / V = df / f = g x / c^2 \qquad (2.1)$$

To understand simply and on a plausibility level the mechanism by which this potential difference around the circuit comes about, we shall analyse the apparatus from the point of view of an observer in an inertial frame. We shall discuss the case where the gravitational field is homogeneous (so that tidal forces can be ignored), and our observer is in free fall, so that he sees the apparatus being accelerated relative to his inertial frame. Let us suppose that at time t the apparatus is at rest, so that a time dt later it is moving with velocity $u = g\,dt$. We shall ignore the necessity of using retarded potentials to describe the electromagnetic field of the system, so that the following analysis is not rigorous.

Consider the simple case where the leads connecting the two weak links are long and parallel, so that they have a uniform capacitance per unit length between them of C_o; from the transmission-line equation the inductance per unit length L_o is given by $C_o L_o c^2 = 1$. The charge per unit length $Q_o = C_o V$ on these wires constitutes a current when it moves with velocity u, which in turn produces a flux threading the loop of:

$$\phi = L_o Q_o u x = (g x / c^2) V dt \qquad (2.2)$$

The average rate of change of flux threading the loop is therefore given approximately by:

$$d\phi/dt = dV = (g\,x / c^2)\,V \qquad (2.3)$$

This is in agreement with the result calculated in equation (2.1).

We now consider the possibility of detecting a gravitational field using modifications of the apparatus described above.

It might be possible to measure a phase difference between the two weak links due to the gravitational field. The photon phase shift will be of order the time for light to cross the apparatus multiplied by the red shift in frequency:

$$d\Theta = df\,x / c = f\,g\,x^2 / c^3 \qquad (2.4)$$

There is however a serious problem with such an apparatus because of difficulties in preventing mechanical strains.* Suppose that the apparatus stretches by a distance dx on account of some external force or thermal expansion. Since the weak links are locked to the electromagnetic radiation, this causes a phase change between the two links of:

$$d\Theta = f\,dx / c \qquad (2.5)$$

If the signal is to be seen above the noise associated with mechanical deformation of the instrument, then comparing (2.5) with (2.4), one obtains the condition:

* A further problem is that it may not be possible to lock the phase across the weak link accurately to the phase of the photons.

$$dx / x < g\ x / c^2 \tag{2.6}$$

Taking values $g = 10\ m\ s^{-2}$, $x = 10\ m$, this would be of order $dx / x = 10^{-15}$. In other words, one would need a dimensional stability upon rotation of the apparatus of order one part in 10^{15} if one is to be able to detect this effect.

It seems impossible to build such an apparatus with present-day materials. For example, using a material with Young's modulus $Y = 10^{11}$ the apparatus must be protected from stresses as low as $10^{-4}\ N\ m^{-2}$ whilst it is being rotated in the gravitational field. Compare with this the gravitational stress of order $\rho\ g\ x$ if the apparatus were constructed using a material with density ρ : using the above values for g and x, together with a density $10^3\ kg\ m^{-3}$, one obtains a stress of $10^5\ N\ m^{-2}$; 9 orders of magnitude too large.

It might be possible to replace the weak links by capacitors, so that gravitationally induced voltage could be detected through the charging currents as the apparatus is rotated in the gravitational field. In this case, no frequency source is required and a high potential difference of order some kilovolts could be used. Using the above values for x and g, the potential difference dV on account of the gravitational field is given by $dV / V = g\ x / c^2 = 10^{-15}$. As above, the dimensional stability required of the apparatus would be of order one part in 10^{15}, so that measurement of this effect does not seem feasible.

Finally, we consider a modification of the capacitance device. In this, the capacitors are replaced by coils with high inductance, and a large current is caused to flow continuously through the circuit. An ammeter is placed in the middle, in parallel with and shorting out the two coils, so that it will detect any current flowing through this short. A simple argument suggests that a current will be caused to flow through the ammeter if the apparatus is placed in a gravitational field. We now give this argument, noting that it is not rigorous; we cannot see a rigorous way to analyse this case.

Our argument involves minimizing the total energy of the system with respect to the current I flowing through the ammeter. A full analysis would take into account the quantization conditions associated with the fluxes in each of the two halves of the circuit, but we do not know how to do this in the presence of a gravitational field.

If each of the coils has inductance L and initially carries current I_o, then when a current $2I$ flows through the ammeter the energies stored in the two coils are given by $2 E_A = L (I_o - I)^2$ in the upper coil, and $2 E_B = L (I_o + I)^2$ in the lower coil. If the system is placed in a gravitational field, then in addition one must take into account the potential energy $E_A\, g\, x / c^2$ associated with the energy of the upper coil. The total energy of the system is therefore given by:

$$2E = L (1 + g x / c^2) (I_o - I)^2 + L (I_o + I)^2 \tag{2.7}$$

Differentiating with respect to I and setting the result equal to zero, one obtains:

$$I = (g x / 2c^2) I_o \tag{2.8}$$

Again, taking the above values for g and x, it would be necessary to have a dimensional stability of order one part in 10^{15}

before this effect could be seen. It appears not to be possible to detect such an effect, although we again emphasize that the above analysis is not rigorous.

3 Superconducting gravity wave detectors

It appears to be possible to build a superconducting apparatus which can detect gravitational radiation. A theoretical analysis of a fluid-based gravity-wave detector has been given in the literature [8], and recently a proposal has been made to detect gravity waves with an apparatus based on superfluid helium-4 [9]. The design to be discussed here is based upon a modification of the concepts described in reference (9).

To understand the nature of gravitational radiation, consider two events which are a vector $x^u = (ct , \underline{x})$ apart in a given co-ordinate frame. The four-distance s between the events is given by:

$$s^2 = x^u g_{uv} x^v \tag{3.1}$$

where the metric is parameterized by the tensor g_{uv} . In an inertial frame with cartesian co-ordinates, g_{uv} has diagonal components (1,-1,-1,-1) and all its other components are zero. The four-distance between the two events is therefore given by $s^2 = c^2t^2 - \underline{x}^2$.

The effect of a gravitational wave passing in the vicinity of the events is to distort the metric g_{uv} , so that:

$$g_{uv} = \mathring{g}_{uv} + h_{uv} \tag{3.2}$$

where the components of h_{uv} are small and they represent the modification of the original metric $\mathring{g}_{uv}$ caused by the wave. The effect of a gravitational wave is therefore to change the distances between the particles and between events.

The passage of a gravity wave does not change directly the momentum of any of the particles in the system. The distance s between events is changed by the wave, but the co-ordinates x^u themselves are not. The momentum $p^u = dx^u / ds$ is therefore unchanged in the chosen co-ordinate frame. It turns out that this co-ordinate frame is the one which has to be used in calculating the gradient of phase of particles using De Broglie's relation (10).

A gravitational wave propagating in the z direction through an object composed of non-interacting particles causes its length to be compressed in the x direction and stretched in the y direction. See figure 2 . The magnitude of these strains varies sinusoidally with time at the frequency f of the radiation, so that the strains reverse direction twice every period. There is a second possible orientation of the gravitational wave, in which the strains are at 45^o to the x axis, within the $x - y$ plane, but we shall not consider this possibility in our simple analysis.

The amplitude of a gravitational wave can be described by a scaler parameter h (not to be confused with Planck's constant). Two noninteracting test particles distance $2r$ apart along the x-axis, subjected to a gravitational wave of amplitude h with the above orientation, will be subjected to a strain

$$dr / r = h \sin(\Omega t) \qquad (3.3)$$

where $\Omega = 2\pi f$.

It might be possible to detect gravitational radiation experimentally. Detectors have been built where the passage of a gravity wave is intended to set up oscillations in an aluminium bar with high quality factor Q . The distance between the ends of the bar is modified during the passage of the wave, and therefore there appear elastic forces which set up the oscillations.

At a frequency of several kilohertz, gravity waves from cosmological sources are believed to bathe the earth which have a duration of a few cycles. The best detectors which have been built at present have a sensitivity of $h = 10^{-18}$ over these few cycles. Gravitational radiation has not yet been detected using them, however (11), so that it is known that events of this magnitude occur at a rate lower than once every few years.

The gravity-wave detector proposed in the literature using superfluid helium-4 is shown schematically in figure 3 . A tube containing the superfluid helium is in the form of a figure-of-eight in the $x - z$ plane with its long axis in the y -direction. The gravitational wave to be detected propagates in the z -direction with wavelength λ , and the width of the apparatus in this direction is $\lambda / 2$.

The effect of the wave is as follows. One long side of the figure-of-eight is subject to compression on account of the wave. Elastic forces in the apparatus oppose this deformation, and are thus used to impart momentum to the superfluid. (One version of the detector would use zeolite molecular sieve material, through which the superfluid would be expected to have a small critical velocity, to effect transfer of momentum from the body of the apparatus to the superfluid.) The momentum would be directed outwards from the centre-line, as shown in the figure. At the other long side, $\lambda / 2$ away, the phase of the wave and hence the direction of deformation is reversed. It can be seen that momentum is imparted to the superfluid always in one direction around the tube. There is thus a circulation $\oint \underline{p}.d\underline{l}$ set up around the tube.

Suppose that the figure-of-eight is broken at one point and the phase difference of the order parameter of the superfluid can be measured across this break. The phase $D\theta$ across the break is given by:

$$D\theta = m^* \oint \underline{u} \cdot d\underline{l} \qquad (3.4)$$

where m^* is the effective mass associated with the order parameter of the superfluid.

The principle behind the proposed method of measuring gravity waves is to insert a detector into the figure-of-eight to measure this phase difference. Although such a detector has never been built successfully, experiments are under way (12) to try to build one using zeolite molecular sieve material. The critical velocity of motion of superfluid through the zeolite is expected to be small, and so the principle is to build the superfluid analogue of a Dayem bridge in superconductivity.

We now consider the possibility of building a gravity-wave detector based upon the superfluid apparatus described above, but using a long superconducting capacitor in the form of a figure-of-eight as shown in figure 4 . The principle of operation resembles that of the superconducting gyroscope (2,3,4), and is now described. As a gravitational wave passes through the figure-of-eight, the fixed charges on the capacitance of the device are set in motion by the elastic forces opposing the deformation due to the wave. The motion of these charges constitutes currents, which flow always in the same direction around the apparatus. A magnetic field is therefore set up. Magnetic induction causes electric currents to flow around the apparatus which tend to null out these fields. These currents are detected by the SQUID.

The time for information to travel around the apparatus to the phase detector must be taken into account in the superfluid helium version. Information cannot travel around the tube faster than the velocity of sound, and so the apparatus should be built with small dimensions and with few turns if it is to be able to detect fast gravity wave signals. This is not as severe a limitation to the superconducting apparatus, however, since information can travel around the tube at the velocity of light.

On the other hand, elastic forces are required to mediate the motion of 'fixed' charge upon the passage of a gravity wave. In the figure there must be a stiff strut across point A to take the forces required to impart momentum in opposite directions to the superconductors on each of the arms of the device. This would provide a limitation to the operating frequency of the device.

In our analysis of the noise limitations of the instrument, we shall ignore the limitations to operating frequency on account of the velocity of sound, and we shall ignore other noise sources such as Brownian motion and external mechanical vibrations. We shall assume that the noise is amplifier-limited. We shall return to these other limitations later.

The velocity of motion of charge generated on account of the gravitational wave in the superconducting apparatus is given approximately by differentiating (3.3) with respect to time. If the apparatus is not one half wavelength across, but instead is distance R across ($R \ll \lambda$), then the relative velocity between the two arms of the device is given by:

$$u \quad = \quad r \; (2 R h \Omega / \lambda) \qquad (3.5)$$

To estimate the sensitivity of such a device to gravitational radiation, let us first consider the limitation due to SQUID noise. It is useful here to compare the gravity wave detector to a superconducting rotation sensor. If a rotation sensor can detect an angular velocity $d\omega$, and it has radius r , then the detectable velocity of motion of charge is of order $u = r\, d\omega$. This can be compared with equation (3.5), which describes the velocity of motion of charge due to the gravity wave. A figure-of-eight gravity-wave detector which is built with similar dimensions and operating parameters as a rotation sensor of sensitivity $d\omega$, would therefore be expected to have a sensitivity to gravity waves of order:

$$d\omega = 2\,R\,dh\,\Omega/\lambda \tag{3}$$

The sensitivity of a rotation device with radius and height 10 cm using a SQUID with energy sensitivity $dE/d\Omega = 5\ 10^{-29}$ J/Hz and with an electric field of 10^{8} V/m is $d\omega\,.\,\tau^{1/2} = 5\ 10^{-6}$ (rad/s) $Hz^{-1/2}$, where τ is the averaging time employed [3]. It is assumed here that an 'ideal' instrument is built with these parameters, so that there are no other sources of noise. This implies a sensitivity to h at a frequency of gravitational radiation f = 10 kHz of:

$$dh.\,\tau^{1/2} = 10^{-5}\ Hz^{-1/2} \tag{3.7}$$

Present-day gravitational wave detectors are capable of detecting a gravity wave parameter $h = 10^{-18}$ over a duration of the wave of a few cycles. If the superconducting apparatus were to be able to compete with such detectors, then taking into account the averaging time of order 10^{-4} seconds, fifteen orders of magnitude improvement would be required.

We now analyse the potential for improving the sensitivity analysed above, on the assumption that amplifier noise is the dominant limitation upon the sensitivity. For the rotation sensor [3], the sensitivity to rotation velocity increases as the square root of the SQUID sensitivity, and it increases as the dimension of the instrument to the 5/2 power. Because of the extra factor R in (3.5), the sensitivity to gravitational radiation would be expected to increase as the dimensions of the instrument to the 7/2 power.

To estimate how far the sensitivity might be improved, it appears to be feasible [13] using present-day technology to improve the SQUID noise by four orders of magnitude over that discussed above. The sensitivity to gravity waves could thus be increased by a factor of 100. If in addition the apparatus were made 10m across, larger by two orders of magnitude, the further gain in sensitivity would be a factor of 10^7. The sensitivity at a frequency of 10 kHz would thus be

$$dh.\tau^{1/2} = 10^{-15}\ Hz^{-1/2} \qquad (3.8)$$

It is common in gravity wave experiments to improve sensitivity by having a high quality factor Q. The passage of a gravity wave is arranged to set up oscillations which take several minutes or hours to decay. The measurement system thus has a longer time τ of several minutes or hours to make its measurement. Greater sensitivity might thus be achieved.

Suppose that the superconducting detector described above were built with an oscillation decay time of 1 hour. Substituting τ = 1 hour into (3.8), one obtains an estimate of the sensitivity to gravity waves:

$$dh = 1.6\ 10^{-17} \qquad (3.9)$$

This is more than an order of magnitude worse than present-day Weber bars. It should be stressed that (3.9) is based on optimistic assumptions and that the practical problems in achieving such sensitivities have not been discussed.

The sensitivity of the device to gravity waves would be expected to increase as the square of the frequency (i.e. the factor Ω/λ in equation (3.5)), so that there is the possibility that higher frequency operation might be useful. One should take into account the velocity of sound as a possible limitation if high frequency performance is envisaged. The flux of radiation from cosmic sources is expected to fall off rapidly with frequency [11], so that it does not appear attractive to take this route.

The sensitivity of the superconducting instrument might be limited in practice by the difficulty of shielding external mechanical vibrations and by brownian motion. The apparatus should be shielded from mechanical strains of order the parameter dh which is to be measured, and this problem has been a major limitation upon the sensitivity of present-day Weber bars. There seems to be no reason to expect that the superconducting apparatus would fare better.

In conclusion, it appears that it is not feasible to build a superconducting gravity wave detector based upon the rotation sensor in a figure-of-eight layout. Even on the most optimistic assumptions about noise limitations, the performance of Weber bars can not be matched with apparatus of reasonable size.

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Acknowledgements

We thank J. Anandan, R.C. Ball, M.W. Cromar, G. Gibbons, B. Muhlfelder, G. Rickayzen, B.D. Josephson, A.B. Pippard, and J.R. Waldram for helpful discussions. The work on the gravity wave detector would not have been possible without patient and clear explanation of gravitational radiation and of the superfluid helium detector by J. Anandan. This work was supported in turn by the Science Research Council, Trinity College, Cambridge and the U.S. Office of Naval Research.

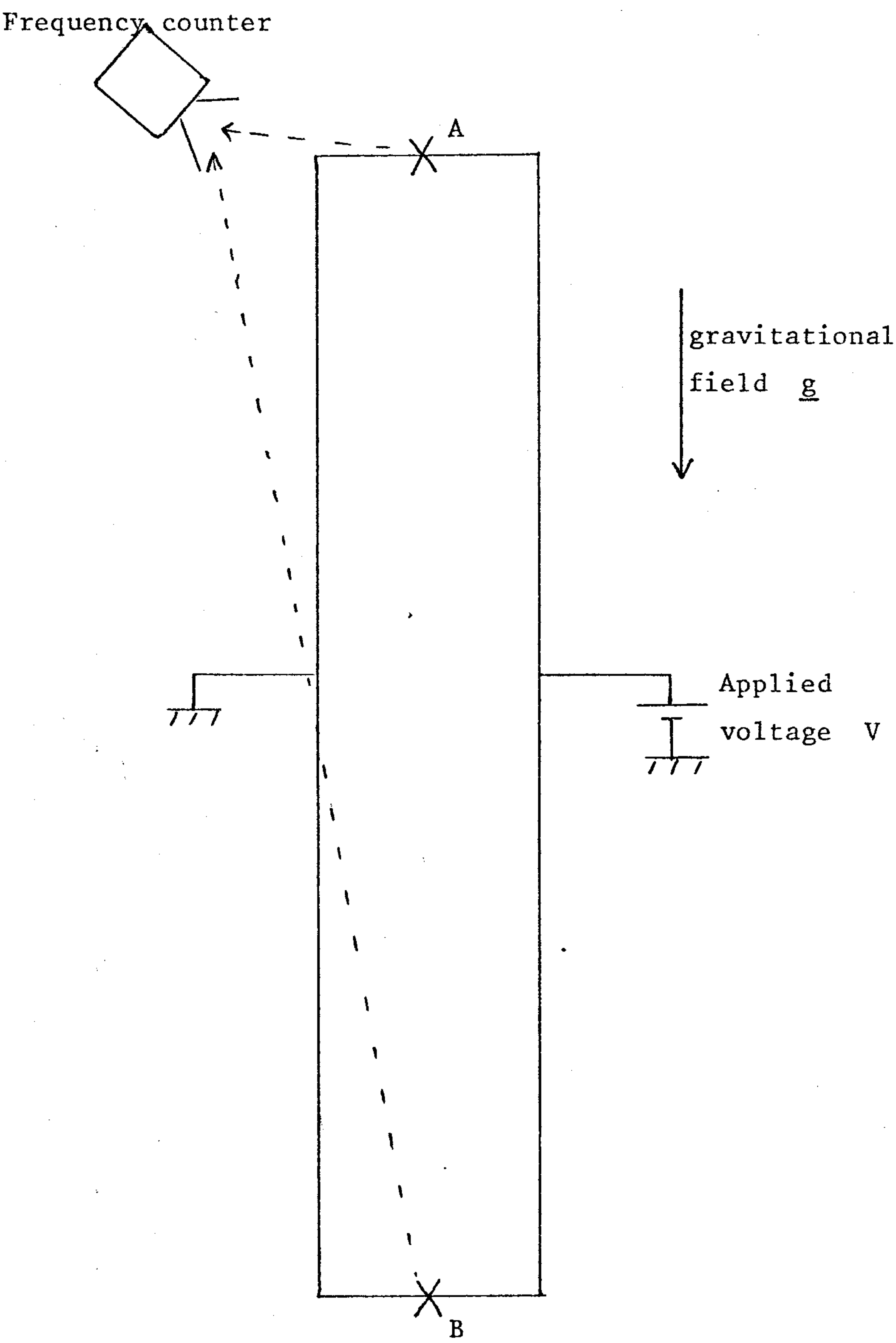
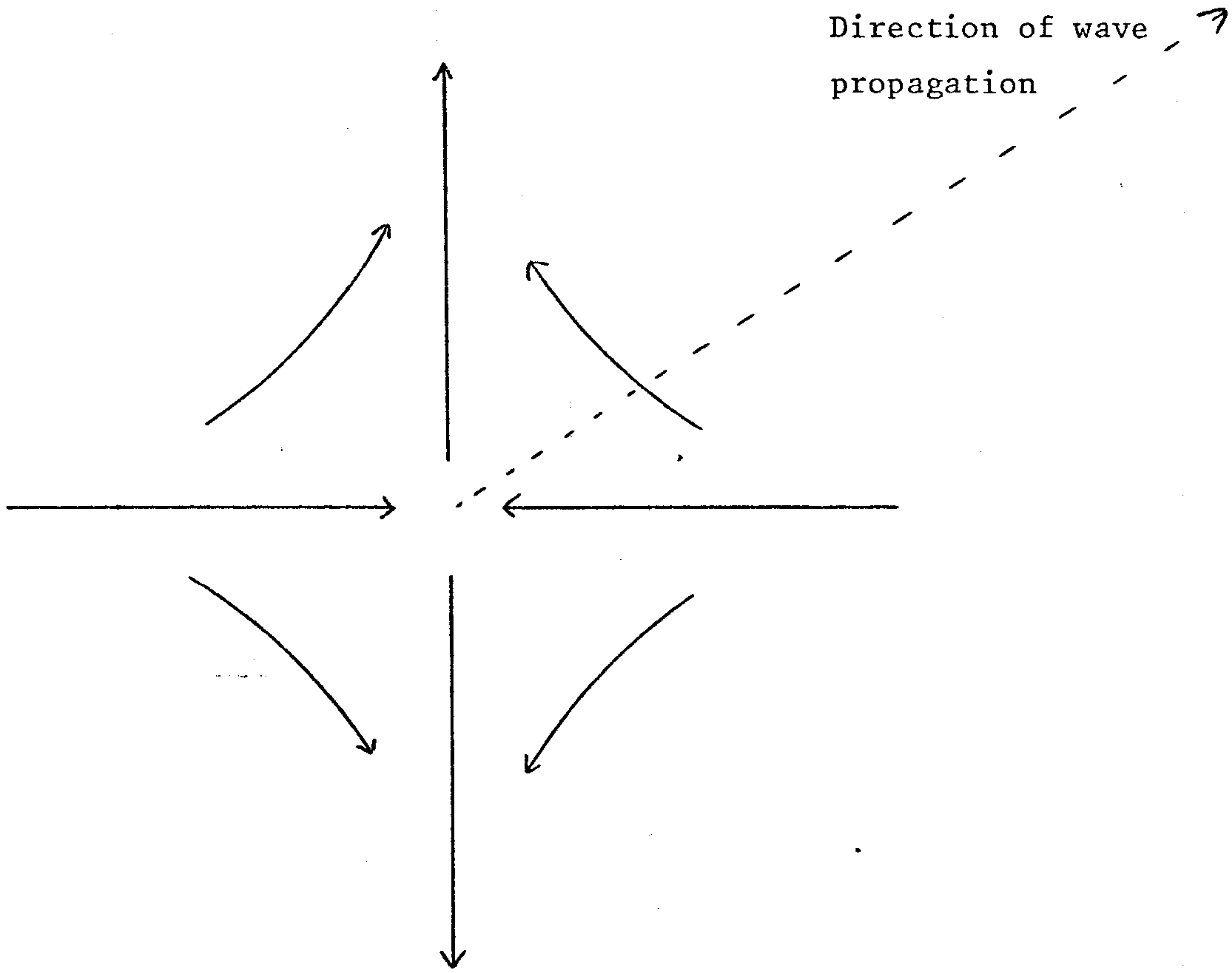


Figure 1 - Proposed superconducting accelerometer



<u>Figure 2</u> - Effect of the passage of a gravitational wave upon the separations of noninteracting particles

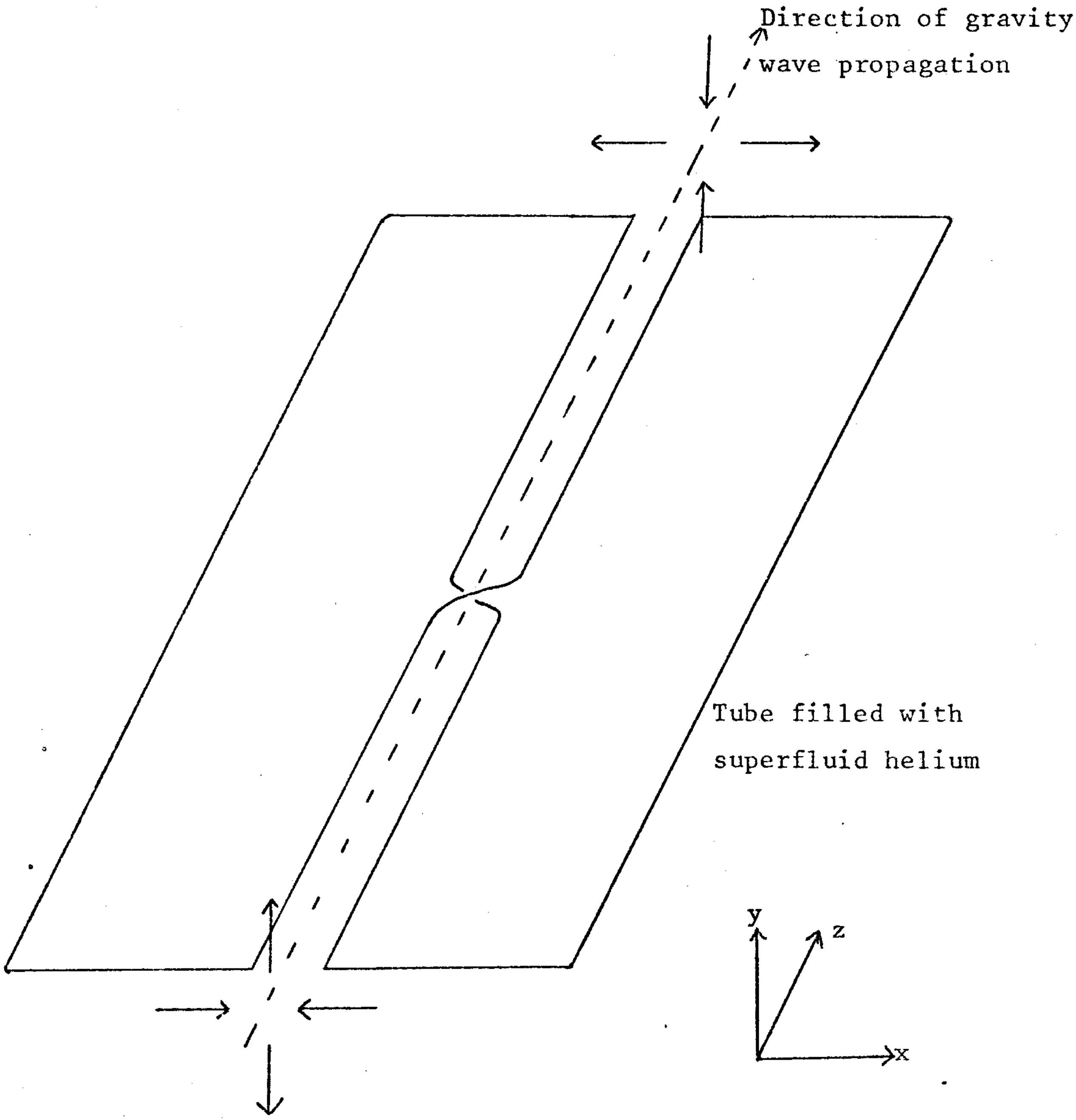


Figure 3 - Geometry of proposed superfluid helium gravity-wave detector

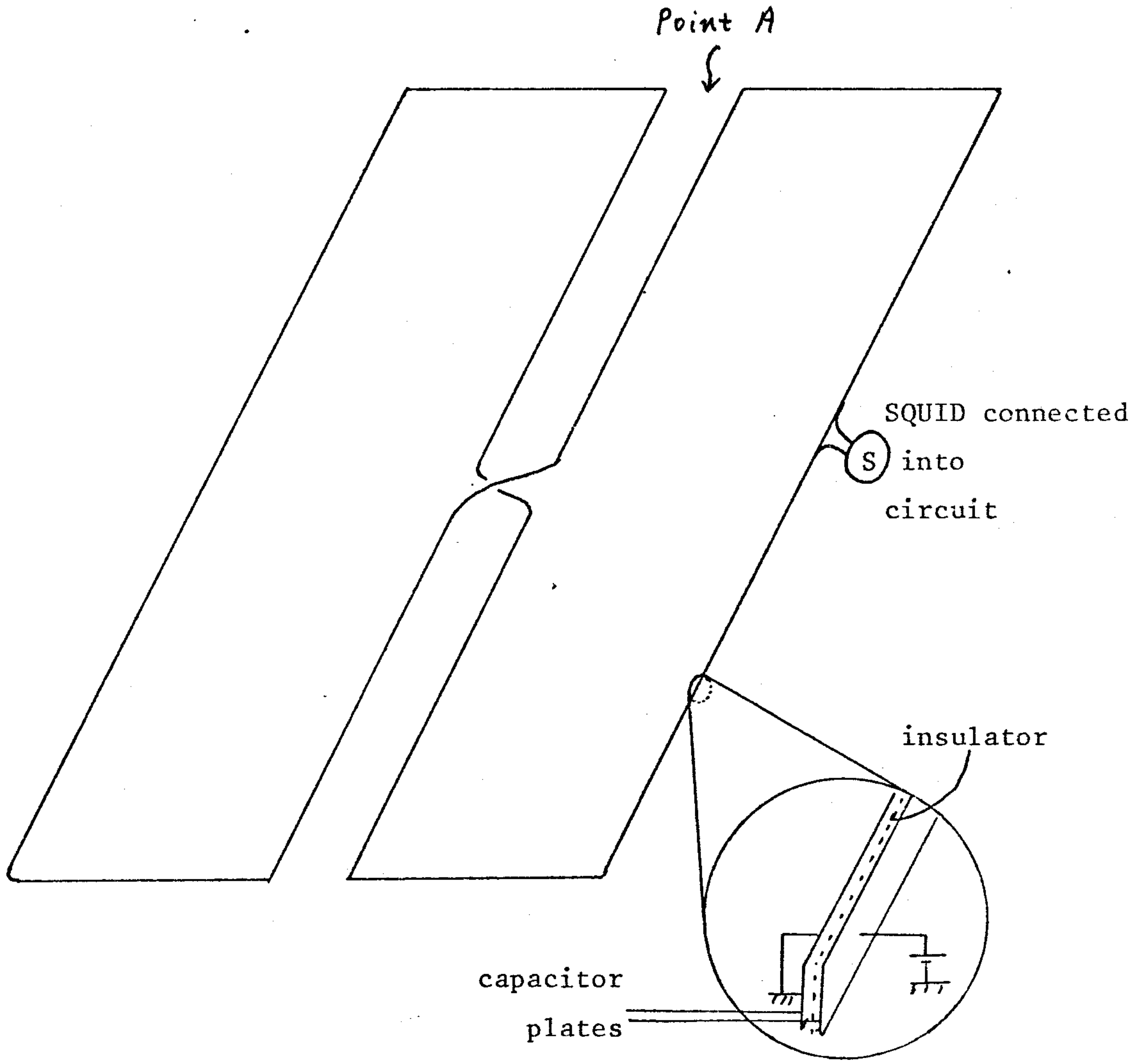


Figure 4 - Superconducting version of the gravity wave detector