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Optimal Exercise of Jointly Held Real Options:
A Nash Bargaining Approach with Value Diversion[*]

Shantanu Banerjee[†]   Ufuk Güçbilmez[‡]   Grzegorz Pawlina[§]

Abstract

This paper provides a two-stage decision framework in which two or more parties exercise a jointly held real option. We show that a single party's timing decision is always socially efficient if it precedes bargaining on the terms of sharing. However, if the sharing rule is agreed before the exercise timing decision is made, then socially optimal timing is attained only if there is a cash payment element in the division of surplus. If the party that chooses the exercise timing can divert value from the project, then the first-best outcome may not be possible at all and the second-best outcome may be implemented using a contract that is generally not optimal in the former cases. Our framework contributes to the understanding of a range of empirical regularities in corporate and entrepreneurial finance.

Keywords: Decision analysis, Investment timing, Real options, Nash bargaining solution, Agency problem

[†]Department of Accounting and Finance, Lancaster University Management School, Lancaster LA1 4YX, UK. Email: `s.banerjee@lancaster.ac.uk`.

[‡]Accounting and Finance Group, University of Edinburgh Business School, 29 Buccleuch Place, Edinburgh EH8 9JS, UK. Email: `u.gucbilmez@ed.ac.uk`.

[§]Corresponding author. Department of Accounting and Finance, Lancaster University Management School, Lancaster LA1 4YX, UK. Email: `g.pawlina@lancaster.ac.uk`. Phone: +44 1524 592834.

1   Introduction

Many real option problems involve two or more parties which can generate a surplus
by jointly exercising an option. In such cases, the option can only be exercised if the
parties agree on the timing of the exercise and on the rule how to divide the proceeds.
For example, when several firms enter into a joint venture to develop an oil field, they
have to agree not only on how they will share revenues when extraction starts, but also
on when to invest in order to start the extraction. Similarly, when biotechnology firms
engage in joint R&D projects, they need to agree on the timing of capital injections
as well as their economic stakes in the projects. Finally, in the context of mergers &
acquisitions, an acquirer and target both care about the timing as well as the terms of
a merger between the two firms.

Some of the real-world scenarios have already attracted attention and are sepa-
rately examined in the literature. Cvitanić, Radas, and Šikić (2011) study optimal
time of entry in the case of a cooperation, such as a joint venture, on a new product
development between a large company and an entrepreneurial firm. Lambrecht (2004)
analyzes a merger between two firms.[1] In this case, the payoff from the option exercise
is the difference between the combined firm value and the sum of the values of individ-
ual firms with no option to merge. The strike price of the option is equivalent to the
sum of fixed (irreversible) costs that each of the firms has to incur when merging. Also,
an expansion of the firm financed by debt (cf. Mauer and Sarkar (2005), Sundaresan
and Wang (2007) and Hackbarth and Mauer (2012)) can be interpreted as a joint real
option exercise. In this case, two parties have to agree on terms of debt repayment,
which directly influences investment timing.

Despite the multitude of situations in which a real option can effectively be jointly
held by two (or even more) parties, a comprehensive analysis of joint exercise strate-
gies has not been undertaken so far. In this paper, we develop a simple yet general
framework that embeds typical contractual arrangements analyzed in the extant liter-
ature as its special cases and derive their efficiency implications for decision making.
We subsequently use the proposed framework to rationalize various types of contracts
observed in economic practice.[2]

We start off by analyzing the optimal exercise policy of a real option that is jointly
held by two parties (firms).[3] In particular, we employ a two-stage decision-making

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[1]Related contributions to the literature on mergers and acquisitions include Morellec and Zhdanov
(2005), Alvarez and Stenbacka (2006), and Lukas, Reuer, and Welling (2012).

[2]We abstain from analyzing a related but distinct situation in which parties *compete* against one
other to be first to exercise a non-exclusive real option. Chevalier-Roignant, Flath, Huchzermeier,
and Trigeorgis (2011) provide a recent comprehensive survey of this stream of literature.

[3]Where appropriate, we use terms "firms" and "parties" interchangeably.

framework in which the parties determine the sharing rule as an outcome of Nash bargaining and one of them makes the exercise decision.[4] In terms of the sharing rule, we consider cash transfers as well as ownership stakes in the project.[5] To capture two different types of contractual arrangements present in the literature, we contrast the scenario in which the exercise decision is made first with the one in which it is the division of proceeds that precedes the exercise decision. We subsequently extend the framework to allow for the real option to be held jointly by any number of parties and demonstrate that the results derived for two firms continue to hold. Finally, to investigate the robustness of earlier results, we analyze a situation in which the party that chooses the investment timing is able to divert a fraction of the project value at a deadweight cost.

We find that when the exercise decision is made first, timing is always socially (and individually) optimal. It is irrelevant which firm makes the investment decision and how bargaining power is distributed among the firms. Furthermore, the result holds even if cash transfers are not allowed for as well as in the case in which the firm that makes the exercise decision simply buys out the stake of the other firm. One special case of this result, with the ratio of the firms' bargaining power coefficients being equal to the ratio of their respective exercise costs, corresponds to the friendly merger discussed in Lambrecht (2004) as well as to Morellec and Zhdanov (2005). If we interpret the model such that one party represents an entrepreneur and the other an investor, the result implies that the entrepreneur always invests optimally regardless of the way he finances the project.

In the opposite case, when the sharing rule is determined first (as, among others, in the hostile takeover scenario of Lambrecht (2004) as well as in Mauer and Sarkar (2005), where a loan commitment is made), investment timing is socially inefficient unless a combination of a stake in the project and a cash transfer is used. In this case, it generally matters which firm makes the exercise decision and what amount of bargaining power it wields. A key implication is that the party exercising the option, e.g., the entrepreneur, is no longer indifferent between the financing choices and may generally invest inefficiently early or late.

We also find that the firm that makes the exercise decision almost always prefers

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[4]We subsequently analyze implications of an assumption that both parties have to agree on the exercise trigger. Moreover, the fact that firms may already receive cash flows from existing assets is irrelevant in Nash bargaining as it is the difference between new and existing cash flows that matters. Therefore the latter can be easily normalized to zero so the solution is interpreted in terms of sharing the *surplus*.

[5]Similar contractual forms are examined in de Bettignies (2008). While this paper focuses on optimal exercise of jointly held real options, de Bettignies focuses on the issue of effort complementarity in a discrete-time setting with no discretion over the investment timing.

to determine the sharing rule first such that it obtains full rights to the project by making a cash transfer to the other firm upon investment, which it inefficiently delays. If that firm is an entrepreneur, this suggests that he almost always prefers (riskless) debt financing where terms are fixed before the investment (debt commitment).[6] We show that equity financing can be rationalized when we extend our framework to allow for the possibility that the entrepreneur can divert part of the project's value at the time of investment.

The present paper studies optimal exercise of jointly held real options in a two-stage decision-making framework. Other contributions featuring such a framework include Shibata and Nishihara (2011) and Lukas, Reuer, and Welling (2012), next to earlier discussed Cvitanić et al (2011), Lambrecht (2004), and Mauer and Sarkar (2005). Shibata and Nishihara (2011) study a setting in which the level of managerial effort is determined first and the exercised decision is made second. In Lukas at al (2012), who study contingent earnouts in mergers and acquisitions, the timing of the takeover is set first, and the target firm only then chooses its level of post-takeover cooperation. Our paper is different from these contributions in that it interacts the sharing rule decision with the decision to exercise the option to invest and the interaction takes place in a broad framework that can be applied to different settings. It also adds to the literature by demonstrating the efficiency implications of the sequence in which the decisions take place, the type of financial contract used, and the value diversion threat.

Bargaining over terms of investment in our framework is comparable to the way buyers and sellers negotiate over terms of trade in a supply chain (Nagarajan and Sošić (2008)). The scenario in which bargaining precedes timing decision is similar to a situation when the wholesale price is fixed first and procurement takes place at a later date (Caldentey and Haugh (2009)). Furthermore, contracts considered in this paper show similarities with those used in studies of coordination in decentralized supply chains. For example, a contract that allows one firm to retain full stake in the project by compensating the other firm with a cash transfer is akin to a wholesale price contract, and the one that combines a stake in the project with a cash transfer is like a revenue sharing contract in which a supplier obtains a share of retail profits by charging a retailer a lower wholesale price. Then, our finding that a combination of a stake in the project and a cash transfer leads to investment efficiency is consistent with the result that revenue sharing contracts can coordinate supply chains (Cachon and Lariviere (2005), Giannoccaro and Pontrandolfo (2004)). The value diversion threat

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[6]As we explain below (riskless) debt financing is equivalent to the entrepreneur making a cash compensation to the investor in return for his input in the project (which may include both monetary as well as a non-monetary component).

in our framework can be likened to the selling effort of the retailer (Taylor (2002) and Gurnani, Erkoc, and Luo (2007)). Finally, two recent papers study optimal time to invest by establishing a supply chain.[7] Chen (2012) analyzes a case in which a supplier and a retailer cooperatively determine the optimal entry time when there is demand uncertainty. Lukas and Welling (2014) model the optimal timing of "climate-friendly" investments in a supply chain. In comparison to these recent contributions, this paper allows a larger contracting set, a value diversion threat and the variation in the sequence of events. Its framework is useful in analyzing contracting problems in the context of mergers and acquisitions, joint ventures, venture capital investments, loan commitments as well as supply chains.

The remainder of this paper is organized as follows. We describe our basic set-up in section 2. The exercise policy is presented in section 3 and the possibility of value diversion is introduced in section 4. Section 5 concludes.

2 Basic set-up

We begin the analysis with a simple case where two parties, i and j, jointly hold a real option to invest in a project. The project requires from each of the parties an irreversible investment outlay of I_i and I_j, respectively (investment outlay may include both monetary as well as a non-monetary contribution). The project value, which is V_t at time t, follows a geometric Brownian motion:

$$dV_t = \alpha V_t dt + \sigma V_t dz_t \tag{1}$$

where α is the constant drift parameter, σ is the constant variance parameter, and dz_t is the increment of a Wiener process.[8] The constant riskless rate is r and parties are risk neutral.[9] The value of the project, V_t, is observable by both parties but not verifiable by a court. This implies that it is impossible to write an enforceable contract that would compel one party to exercise the option at a particular level of V_t.

Without a loss of generality, the control right over the exercise decision is allocated to firm i. However, firm i has to agree with firm j over the sharing rule of the project's

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[7]Other type of real options (or flexibilities) in supply chains include reordering and return options (Wu and Kleindorfer (2005), Burnetas and Ritchken (2005)), option to switch supplier (Kamrad and Siddique (2004)), and flexibility to relax the retailer's budget constraint (Caldentey and Haugh (2009)).

[8]Throughout the rest of the paper we drop the subscript t, and denote V_t as V for the sake of simplicity.

[9]Alternatively, one could assume that the payout from the project is spanned by a portfolio of traded assets.

value. In other words, while the timing of investment is determined by one party, the terms of investment is determined by both parties.

The sharing rule specifies the fraction of the project value received by each firm. firm i receives fraction $\gamma_i \geq 0$ of the project (so $\gamma_i + \gamma_j = 1$). We also allow for fixed payments between firms, such that the amount firm i pays to firm j is C_i (so $C_i = -C_j$). Quantities γ_i and C_i are endogenously determined as outcomes of Nash bargaining. The bargaining power parameter of firm i is given by $\eta_i \geq 0$ (with $\eta_i + \eta_j = 1$).[10] In general, when the investment occurs, firm i receives the net payoff equal to $\gamma_i V - C_i - I_i$.

One scenario that corresponds to this setting is when firm i is a wealth-constrained entrepreneurial firm and firm j is an investor (cf. Jørgensen, Kort, and Dockner (2006)). The entrepreneurial firm owns the project, but needs outside financing from the investor, who may, for example, be a venture capitalist. If the only role of the investor was to provide financing, perfectly competitive capital markets would imply that $\eta_i = 1$ in such a case.[11] Another relevant scenario is when firms i and j are two firms (potentially in the same industry) that have a real option to start a joint venture. We refer to these two scenarios as the 'entrepreneurial finance (EF) scenario' and 'joint venture (JV) scenario' respectively.

2.1 The project's capital and ownership structure

The outcome of bargaining can lead to various types of capital structure. γ_i can be thought of as firm i's equity claim on the project. C_i can be either a cash transfer from firm i to j, or the value of riskless debt issued by firm i and held by firm j. In the latter case, $C_i = \frac{b_i}{r}$, where b_i is the perpetual coupon flow that firm i pays to firm j in exchange for firm j's initial contribution to the project, I_j. Consequently, once the investment takes place, the project's capital structure can take a few different forms.

Table 1 summarizes the project's capital and ownership structures under the scenarios EF and JV and for different levels of γ_i and C_i. The first possible capital structure involves no cash transfers $C_i = 0$ (which essentially implies that $0 < \gamma_i < 1$). In the EF scenario, this corresponds to a case in which the entrepreneur uses pure equity financing. In the JV scenario, it implies that firms share the project's ownership without any side payments. The second possible capital structure features full ownership by

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[10]We assume that the bargaining power distribution is exogenous and cannot be influenced by, e.g., the choice of capital structure (as in Perotti and Spier (1993) and Lambrecht and Pawlina (2013), see also the discussion in Banerjee, Dasgupta, and Kim (2008)).

[11]When there is perfect competition for financing projects, investors are ready to invest in projects that are expected to yield them zero net payoffs. This is equivalent to the entrepreneurial firm having the full bargaining power (i.e., $\eta_i = 1$), since the entrepreneur appropriates the entire net present value of the project, while the investor merely expects to break even.

one of the two firms, such that either $\gamma_i = 1$ (which implies $C_i > 0$) or $\gamma_i = 0$ (which implies $C_i < 0$). In the context of the EF scenario, the entrepreneur is using pure debt financing when $\gamma_i = 1$ or cashing out by selling the project to the investor when $\gamma_i = 0$. In terms of the JV scenario, firm i is obtaining the project's full ownership when $\gamma_i = 1$, or ceding it when $\gamma_i = 0$.[12] Finally, a third possibility involves cash transfers alongside a share of ownership, such that $0 < \gamma_i < 1$ and $C_i \neq 0$. For the EF scenario, this implies that the entrepreneur uses a mixture of equity and debt financing. For the JV scenario, it implies that firms engage in side payments along with sharing the project's ownership.

[Insert Table 1 about here]

2.2 The sequence of moves

In our analysis, we allow not only for the the possibility of sharing the surplus and cash transfers, but also allow for two different sequences of moves, as they have been demonstrated to affect the option exercise policy (see, e.g., Lambrecht (2004)). In particular, we consider two situations: (1) firm i choosing the timing of the exercise and subsequently firms i and j choosing the terms (using the Nash bargaining solution), or (2) firms i and j committing to the terms and only then firm i choosing the timing.

These two different sequences of moves are encountered in various different settings in the literature. Lambrecht (2004) studies them in the context of mergers and acquisitions. The former sequence in his model corresponds to a friendly merger in which the timing of the merger is agreed first and then the terms of the merger are determined. The latter sequence corresponds to a hostile takeover such that the target commits to the terms and the acquirer only then chooses the timing of takeover. Similarly, Cvitanić et al. (2011) study them in the context of joint ventures allowing one firm to choose the timing of venture while the other sets the terms. The latter sequence is studied by Mauer and Sarkar (2005) in the context of debt financing in a perfectly competitive capital market. In Mauer and Sarkar (2005), equityholders of a firm first agree with debtholders the terms of a loan commitment, and then once these terms are fixed, equityholders decide when to invest. This is in contrast to, for instance, Sundaresan and Wang (2007), where terms of debt financing are effectively set once the choice of investment timing is known.

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[12]In our subsequent analysis, we do not consider the case when $\gamma_i = 0$, since its outcome is trivial. It is optimal for firm i to invest at V_0 (i.e., immediately at $t = 0$) when it cedes its right to the project in return for a cash transfer from firm j.

We demonstrate that the sequence of moves has a profound effect on the timing of the option exercise, which, in turn, has significant value implications. The reason why the sequence of moves matters is as follows. If firm i first chooses the threshold, and then bargains with firm j ("exercise decision first, sharing rule second"), the investment timing decision is ex ante and is shown to maximize the total value of the investment option. If firm i first bargains with firm j, and then chooses the threshold ("sharing rule first, exercise decision second"), the investment timing decision is ex post and it is only the value of firm i's claim that is maximized. In the former case, firm i chooses the investment timing in anticipation of the outcome of the bargaining game. In the latter case, firms play the bargaining game in anticipation of firm i's subsequent choice of investment threshold.[13]

3 Exercise policy

3.1 The efficient outcome

In order to evaluate whether the actions of firms lead to an efficient outcome, we determine the first-best investment threshold. The first-best threshold maximizes the expected net present value (NPV) of the project, defined as the NPV of the project at the time of investment multiplied by the discount factor reflecting the present value of \$1 received upon hitting the investment threshold (see (4) below). This threshold solves the following problem (see Dixit and Pindyck (1994), ch. 5):

$$\max_V (V - I) \left(\frac{V_0}{V}\right)^\beta \tag{2}$$

and is equal to:

$$\overline{V} = kI \tag{3}$$

where $I = I_i + I_j$, $k = \frac{\beta}{\beta-1}$ and $\beta = \frac{1}{\sigma^2}(-(\alpha - \sigma^2/2) + \sqrt{(\alpha - \sigma^2/2)^2 + 2r\sigma^2})$. The project's expected NPV is maximized if firm i exercises the real option when the project value reaches $\overline{V}$. The maximum expected NPV is equal to:

$$\overline{NPV} = (\overline{V} - I) \left(\frac{V_0}{\overline{V}}\right)^\beta = (k - 1)I \left(\frac{V_0}{kI}\right)^\beta \tag{4}$$

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[13]When $\eta_i = 0$, both situations can be thought of as different types of Stackelberg games. In particular, when the exercise decision is made first (sharing rule is determined first), firm i chooses the investment threshold as a leader (follower), and firm j determines the sharing rule as a follower (leader). When $\eta_i = 1$, firm i dictates both the timing and the terms of investment, and firm j simply breaks even.

If firm i chooses a lower (higher) threshold, the expected NPV is reduced due to investing inefficiently early (late). We assume that V_0 is sufficiently low so that immediate investment is not optimal.

3.2 Exercise decision first, sharing rule second

The firms can agree to exercise the option by choosing the timing of the option exercise first and only subsequently agree on how to split the surplus. Initially, firm i decides when to invest, so that it chooses the investment threshold that maximizes the value of its option to invest. When the project value reaches that threshold, firm i bargains with firm j over the terms of financing. If they agree, they invest in the project and split the proceeds as agreed. Alternatively, firm i can make a payment of C_i to firm j as a compensation for incurring cost I_j and acquire the entire project V. In the former case, the firms bargain over the equity stake γ_i, whereas in the latter case, the magnitude of payment C_i has to be agreed upon. In either case, the result of bargaining depends on the investment threshold chosen by firm i in advance. Obviously, firm i anticipates the sharing rule that will be agreed at any threshold (i.e. $\gamma_i(V)$, $C_i(V)$ or a combination thereof), and chooses V accordingly. For two firms i and j, we have $\gamma_j = 1 - \gamma_i$, $C_j = -C_i$, $\eta_j = 1 - \eta_i$ and $I_j = I - I_i$. Thus, the bargaining problem is:

$$\max_{\gamma_i, C_i} \left[\gamma_i V - C_i - I_i\right]^{\eta_i} \left[(1 - \gamma_i)V + C_i - (I - I_i)\right]^{1 - \eta_i} \tag{5}$$

Firm i makes the exercise decision given the anticipated outcome of bargaining $\gamma_i(V)$ and $C_i(V)$:

$$\max_{V} \left[\gamma_i(V)V - C_i(V) - I_i\right] \left(\frac{V_0}{V}\right)^{\beta} \tag{6}$$

Proposition 1 (Exercise decision first, sharing rule second)

(i) $C_{e,i} = 0$: If the exercise timing decision is made first and the sharing rule is determined second, the fraction of the project's value received by firm i is:

$$\gamma_{e,i} = \frac{I_i + \eta_i(k-1)I}{kI} \tag{7}$$

(ii) $\gamma_{e,i} = 1$: If firm j is compensated by firm i in return for fully ceding its rights to the project, it receives payment C_i equal to:

$$C_{e,i} = I_j + \eta_j(k-1)I \tag{8}$$

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(iii) $0 < \gamma_{e,i} < 1$ *and* $C_{e,i} \neq 0$: *Finally, the firms can agree to split the project between them and provide an additional compensation in cash. In such a case, the fraction of the project retained by firm i and its cash compensation satisfy the following equation:*

$$\gamma_{e,i} kI + C_{e,i} - I_i = \eta_i (k-1) I \tag{9}$$

In all three cases, the investment threshold is:

$$\overline{V}_e = \overline{V} \tag{10}$$

Proof. See Appendix A ∎

Equations (7) and (8) indicate that the stake in the project $\gamma_{e,i}$ increases and the amount of cash compensation $C_{e,i}$ decreases with firm i's bargaining power coefficient η_i and its contribution to the project I_i. Since $\partial \gamma_{e,i}/\partial k \gtrless 0$ for $\eta_i \gtrless I_i/I$, the received fraction of the project increases with project volatility (as well as its growth rate and decreases with the interest rate) if firm i's bargaining power is sufficiently high and decreases otherwise.[14] Moreover, $\partial C_{e,i}/\partial k > 0$ (cf. (8)), so the amount of cash compensation increases with project volatility irrespective from the value of the bargaining power coefficient.

When the exercise decision is made first, and the sharing rule is determined second, the investment timing is always optimal (i.e. $\overline{V}_e = \overline{V}$), regardless of the distribution of bargaining power and of the choice between equity and cash compensation. The intuition is as follows. When firm i is making the exercise decision, it anticipates that it will get a fixed slice of the surplus regardless of when the investment takes place. Therefore, it is optimal for firm i that the investment takes place when the surplus is maximized, since this also maximizes the value of firm i's slice of the surplus. In fact, if it were firm j making the exercise decision, it would choose the same investment threshold, since, like firm i, firm j anticipates that it will receive a fixed fraction of the surplus and therefore it would also have an incentive to maximize the surplus. As a result, in the analyzed scenario it is irrelevant which firm chooses the timing (or, whether they do it jointly) as each firm has an incentive to choose the first-best investment threshold.[15]

Proposition 1 has specific implications for the EF and JV scenarios. In terms of the EF scenario, the entrepreneur is indifferent between financing the project with equity, debt, or a mixture of both. In other words, the entrepreneur's investment

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[14]Note that $\partial k/\partial \sigma > 0$, $\partial k/\partial r < 0$ and $\partial k/\partial \alpha > 0$, cf. Dixit and Pindyck (1994), ch. 5.

[15]The (friendly merger) outcome in Lambrecht (2004) (see also Morellec and Zhdanov (2005)) is a special case of the situation described in Proposition 1 with $\gamma_i = \eta_i = I_i/I$.

timing decision can be separated from his financing decision, and the capital structure is irrelevant. In terms of the JV scenario, the allocation of control right over the exercise decision is irrelevant. Each firm chooses the same investment threshold when it has the control right. In other words, firms i and j agree on when to exercise their joint real option to form a joint venture.

3.3 Sharing rule first, exercise decision second

One could easily envisage a situation where firms first commit to the terms of a deal and only subsequently choose its timing (cf. Mauer and Sarkar (2005)). In such a case, at $t = 0$ firms bargain with each other either about the equity stake γ_i or the cash payment C_i, or both. If the parties agree, they commit to provide I_i and I_j at the time of investment. The key point is that, after the terms of investment is set, firm i's option to delay investment is still alive. In general, and unlike in the previous subsection, it matters which firm chooses the timing of investment once the terms are agreed upon. Obviously, the investment threshold depends on the financial terms that the firms have been agreed upon. If the parties are negotiating the sharing rule without cash transfers, once γ_i is fixed, firm i will maximize the expected NPV based on its share of the project value $\gamma_i V$. Similarly, if they are negotiating the cash compensation associated with firm j ceding its right to the project, once C_i is fixed, firm i will maximize the expected NPV based on its residual claim $V - C_i$. Therefore, the parties bargain over financial terms, taking into account firm i's ex-post choice of investment threshold (i.e. $V(\gamma_i)$, $V(C_i)$, or $V(\gamma_i, C_i)$). Firm i's exercise decision is:

$$\max_V (\gamma_i V - C_i - I_i)\left(\frac{V_0}{V}\right)^\beta \tag{11}$$

The parties bargain anticipating that firm i will invest when the project value reaches $V(\gamma_i, C_i)$:

$$\max_{\gamma_i, C_i} \frac{[\gamma_i V(\gamma_i, C_i) - C_i - I_i]^{\eta_i}}{[(1 - \gamma_i)V(\gamma_i, C_i) + C_i - (I - I_i)]^{\eta_i - 1}}\left(\frac{V_0}{V(\gamma_i, C_i)}\right)^\beta \tag{12}$$

Proposition 2 (Sharing rule first, exercise decision second)

(i) $C_{s,i} = 0$: *If the sharing rule is determined first and the timing decision is determined by firm i second, the fraction of the project's value received by firm i and*

the investment threshold are:

$$\gamma_{s,i} = \frac{I_i[1 + \eta_i(k-1)]}{I + (k-1)I_i} \tag{13}$$

$$\overline{V}_s^{(\gamma)} = \frac{1 + (I_i/I)(k-1)}{1 + \eta_i(k-1)}\overline{V} \tag{14}$$

(ii) $\gamma_{s,i} = 1$: If firm j is compensated by firm i in return for fully ceding its rights to the project, cash transfer C_i and the investment threshold equal

$$C_{s,i} = I_j + \eta_j(k-1)I \tag{15}$$

$$\overline{V}_s^{(C)} = (1 + (1-\eta_i)(k-1))\overline{V} \tag{16}$$

(iii) $0 < \gamma_{s,i} < 1$ and $C_{s,i} \neq 0$: If a combination of a stake in the project and cash transfers is allowed, the equity stake, cash transfer and the investment threshold are:

$$\gamma_{s,i} = \eta_i \tag{17}$$

$$C_{s,i} = \eta_i I - I_i \tag{18}$$

$$\overline{V}_s^{(\gamma,C)} = \overline{V} \tag{19}$$

Proof. See Appendix A ∎

As in the previous case, the stake in the project $\gamma_{s,i}$ increases and the amount of cash compensation $C_{s,i}$ decreases with both firm i's bargaining power, η_i, and its contribution to the project, I_i. It also holds that $\partial\gamma_{s,i}/\partial k \gtrless 0$ for $\eta_i \gtrless I_i/I$, so the fraction of the project received increases with its volatility if firm i's bargaining power is sufficiently high. Moreover, cash compensation $C_{s,i}$ unambiguously increases with project volatility (cf. (15)).

In the absence of cash transfers, the investment threshold chosen by firm i decreases as its bargaining power increases (i.e., $\partial\overline{V}_s^{(\gamma)}/\partial\eta_i < 0$). The reason is as follows. When η_i is low, the firm's equity stake $\gamma_{s,i}$ is also low, so in order to receive a sufficiently large payoff, the firm delays investment. On the other hand, when η_i is high, the firm's equity stake $\gamma_{s,i}$ is also high, so in order to realize the large payoff sooner, the firm hurries investment. The investment timing is always privately optimal for the firm ex post, but is generally socially suboptimal, since $\overline{V}_s^{(\gamma)} \neq \overline{V}$. Only in the knife-edge case of $\eta_i = I_i/I$, the delay is both privately and socially optimal. Consequently, in the absence of cash transfers, a deal in which the decision how to split the surplus precedes

the choice of the investment timing leads to an inefficiently timed option exercise. In general, firm i invests too late when its bargaining power is low, and too early when its bargaining power is high.[16] Finally, the direction of the effect of volatility (via k) on the investment delay $(\overline{V}_s^{(\gamma)}/\overline{V})$ depends on the distribution of bargaining power. It holds that $\partial(\overline{V}_s^{(\gamma)}/\overline{V})/\partial k \lesseqgtr 0$ for $\eta_i \gtreqless I_i/I$, which reflects the effect of k on the equity stake: an increasing (decreasing) stake $\gamma_{s,i}$ in response to an increase in volatility results in a lower (higher) investment threshold $\overline{V}_s^{(\gamma)}$ (relative to $\overline{V}$) due to investment becoming more (less) attractive.

When firm i compensates for the contribution of the other part fully in cash, the investment threshold increases when firm i's bargaining power decreases $(\partial\overline{V}_s^{(C)}/\partial\eta_i <$ 0. In other words, firm i delays investment longer when its bargaining position is weaker. The rationale for this is as follows. The cash payment $C_{s,i}$ is higher when the firm i's bargaining position is weaker $(\partial C_{s,i}/\partial\eta_i < 0)$. And when $C_{s,i}$ is higher, firm i's net payoff at the time of investment $V - C_{s,i} - I_i$ is lower. Consequently, firm i has to delay investment longer until it receives a sufficiently high payoff. The duration of the delay is privately optimal for firm i, but for $\forall\eta_i < 1$ it is socially suboptimal $(\overline{V}_s^{(C)} > \overline{V})$. As $\partial(\overline{V}_s^{(C)}/\overline{V})/\partial k > 0$, higher volatility unambiguously exacerbates the investment delay. Only when $\eta_i = 1$, this duration is both privately and socially optimal. Therefore, in general, allowing for cash-only compensation with bargaining taking place before the option is exercised leads to inefficiently late investment.

The last part of Proposition 2 describes the consequences of a cash transfer $C_{s,i}$ between firms when they bargain over the sharing rule $\gamma_{s,i}$. Unlike in the previous two cases $(C_{s,i} = 0$ and $\gamma_{s,i} = 1)$, in this case firm i always chooses the first-best investment threshold. In other words, inefficiencies in investment timing observed in previous two cases disappear when parties bargain over $C_{s,i}$ as well as $\gamma_{s,i}$. The intuition is as follows. The overall impact of η_i on $\overline{V}_s^{(\gamma,C)}$ can be decomposed into two parts:[17]

$$\frac{d\overline{V}_s^{(\gamma,C)}}{d\eta_i} = \frac{\partial\overline{V}_s^{(\gamma,C)}}{\partial\gamma_{s,i}}\frac{\partial\gamma_{s,i}}{\partial\eta_i} + \frac{\partial\overline{V}_s^{(\gamma,C)}}{\partial C_{s,i}}\frac{\partial C_{s,i}}{\partial\eta_i}$$

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[16]The extreme form of former case ($\eta_i = 0$) corresponds to the hostile takeover in Lambrecht (2004). In that situation the target firm chooses the stake in the joint firm that it requires to receive and the raider chooses the timing of consummating the deal.

[17]Siddiqui and Takashima (2012) employ a similar approach when analyzing capacity switching decisions in a non-preemptive duopoly.

When η_i increases, firm i receives a higher equity stake, which incentivizes the firm to accelerate investment:

$$\frac{\partial \overline{V}_s^{(\gamma,C)}}{\partial \gamma_{s,i}} \frac{\partial \gamma_{s,i}}{\partial \eta_i} = -\frac{k(I_i + C_{s,i})}{\gamma_{s,i}^2} < 0$$

At the same time, the increase in η_i implies that firm i has to make a larger cash payment to firm j. From firm i's perspective, the larger cash payment is like an additional fixed cost of investment, which incentivizes the firm to delay investment:

$$\frac{\partial \overline{V}_s^{(\gamma,C)}}{\partial C_{s,i}} \frac{\partial C_{s,i}}{\partial \eta_i} = \frac{kI}{\gamma_{s,i}} > 0$$

When $\gamma_{s,i} = \eta_i$ and $C_{s,i} = \eta_i I - I_i$, these contrarian incentives cancel out each other, such that it is always optimal for firm i to choose the first-best investment threshold regardless of its bargaining power η_i:

$$\left. \frac{d\overline{V}_s^{(\gamma,C)}}{d\eta_i} \right|_{\gamma_{s,i}=\eta_i,\ C_{s,i}=\eta_i I - I_i} = -\frac{k(I_i + \eta_i I - I_i)}{\eta_i^2} + \frac{kI}{\eta_i} = 0$$

Like Proposition 1, Proposition 2 also has specific implications for the EF and JV scenarios. In terms of the EF scenario, we can no longer separate the entrepreneur's investment timing decision from his financing decision, since the investment threshold various across the financing choices, and the entrepreneur is no longer indifferent between those choices. We examine this further in section 3.4. In terms of the JV scenario, it is no longer guaranteed that each firm will have an incentive to invest at the same time. That is, unless a combination of a stake in the project and cash transfers is used, firms will be likely to disagree on the timing of entry to a joint venture. This issue is discussed further in section 3.5.

3.4   Firm i's choice

We have so far examined the timing and terms of investment under different contractual forms in Propositions 1 and 2. In this section, we investigate firm i's choice between these contractual forms. This choice depends on firm i's expected NPV, which is determined by its claim (net of investment) on the project and the investment threshold $\overline{V}_m$:

$$NPV_{m,i} = (\gamma_{m,i}\overline{V}_m - C_{m,i} - I_i)\left(\frac{V_0}{\overline{V}_m}\right)^\beta \tag{20}$$

where $m \in \{e, s\}$. We define $NPV_{e,i}$ as firm i's expected NPV when the exercise decision is made first. Using equations (7)-(10) presented in Proposition 1, it is easy to show that $NPV_{e,i}$ is always equal to $\eta_i(k-1)I(V_0/kI)^\beta$ regardless of whether cash transfers are involved or not. We also define $NPV_{s,i}$ as firm i's expected NPV when the sharing rule is determined first. Unlike $NPV_{e,i}$, the value of $NPV_{s,i}$ changes when only equity stakes or cash transfers are involved. Therefore, we use the notation $NPV_{s,i}^{(C)}$ when firm i acquires the full rights to project such that $\gamma_i = 1$, and $NPV_{s,i}^{(\gamma)}$ when cash transfers are not allowed such that $C_i = 0$. Finally, the notation $NPV_{s,i}^{(\gamma,C)}$ implies that a combination of a stake in the project and cash transfers is used.

Proposition 3 (Firm i's choice) *Firm i is indifferent between making the exercise decision first versus determining the sharing rule first, such that in the latter case the sharing rule involves a combination of a stake in the project and cash transfers:*

$$NPV_{e,i} = NPV_{s,i}^{(\gamma,C)} \tag{21}$$

For $0 \leq \eta_i < 1$, firm i prefers to determine the sharing rule first and obtain full rights to the project at the time of investment:

$$NPV_{s,i}^{(C)} > \max\left(NPV_{s,i}^{(\gamma,C)}, NPV_{s,i}^{(\gamma)}\right) \tag{22}$$

For $\eta_i = 1$, when the sharing rule is determined first, firm i is indifferent between obtaining full rights to the project at the time of investment versus sharing the project's ownership with a cash transfer, and prefers these two options over having no cash transfers:

$$NPV_{s,i}^{(C)} = NPV_{s,i}^{(\gamma,C)} > NPV_{s,i}^{(\gamma)} \tag{23}$$

Proof. See Appendix A ■

Proposition 3 is illustrated in Figure 1 for the following set of parameter values: $I_i = 25$, $I_j = 75$, $V_0 = 100$, and $\beta = 2$. When the sharing rule is decided first, firm i almost always prefers to acquire full rights to the project and invests inefficiently late (see the solid lines in panels (a) and (b)). Only when $\eta_i = 1$, it is indifferent between this option and sharing the project's ownership with firm j and having a cash transfer, in which case firm i's investment policy is socially optimal. Interestingly, according to Proposition 3, firm i *prefers* to decide the sharing rule first, since its expected NPV in this case is always higher compared to the case when the exercise decision is made first, except when $\eta_i = 1$, in which case it is indifferent.

[Insert Figure 1 about here]

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We interpret Proposition 3 in the context of the EF scenario as follows. The entrepreneur almost always prefers debt financing with terms fixed first. Only when he has full bargaining power he is indifferent between this option and mixed financing with terms first (i.e., the last option in Proposition 2) or any type of financing with timing fixed first (i.e., the options in Proposition 1).

The intuition behind the entrepreneur's choice is as follows. When the exercise decision is made first, the entrepreneur kills his option to invest before bargaining, whereas when the sharing rule is decided first, his option to invest is still alive after bargaining. In the former case, the entrepreneur has an incentive to maximize the project's expected NPV, since he anticipates that he will receive a fixed fraction of the project's NPV. In the latter case, since bargaining has already taken place, he has an incentive to maximize the expected NPV of his claim on the project. In other words, there is an agency problem (see Jensen and Meckling (1976)) when the entrepreneur makes the investment timing decision ex post.[18]

It is worth emphasizing that the entrepreneur's dominant choice is pure debt financing, equivalent to a commitment to paying the present value of C_i to the financier, with terms fixed first. He never prefers pure equity financing (except for the case when $\eta_i = 1$ and timing being fixed first, when he is indifferent between any combination of debt and equity financing). As it stands there is no demand for pure equity financing in this setting. We return to this point in section 4 when we consider the possibility that the entrepreneur can divert part of the project's value.

3.5   Alignment of investment incentives with the sharing rule

When the exercise decision is made first, Proposition 1 suggests that each firm has an incentive to invest at the same time, which is also the socially optimal timing of investment. However, when the sharing rule is determined first, according to Proposition 2, firms will generally not agree on investment timing, especially when a combination of a stake in the project and cash transfers cannot be used. This is problematic particularly for the JV scenario, since the joint nature of the real option to start a joint venture requires that firms agree on timing as well as terms of investment. One way to align investment timing incentives is to design a sharing rule with effectively an endogenous bargaining power distribution which gives each firm an incentive to invest at the same time, as in Morellec and Zhdanov (2005).

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[18]Note that a suboptimal investment policy arises despite the fact that there is no asymmetric information between the entrepreneur and the investor (cf. Mæland (2002)and Grenadier and Wang (2005)), and when the entrepreneur is risk neutral (cf. Hugonnier and Morellec (2007) and Chronopoulos, De Reyck, and Siddiqui (2011)).

Proposition 4 (Alignment of investment incentives with the sharing rule)

i and j have incentives to invest at the same time when the sharing rule satisfies:

$$\gamma_{s,i} = \frac{I_i + C_{s,i}}{I} \tag{24}$$

The jointly-agreed investment threshold is always $\overline{V}$.

Proof. See Appendix A ∎

Proposition 2 showed that when the sharing rule is decided first with Nash bargaining, each firm adopts the socially optimal investment policy only if a combination of a stake in the project and cash transfers is used (i.e., $0 < \gamma_{s,i} < 1$ and $C_{s,i} \neq 0$). This is consistent with Proposition 4, since $\gamma_{s,i}$ and $C_{s,i}$ given in equations (17) and (18) satisfy equation (24). If no cash transfers are allowed (i.e., $C_{s,i} = 0$), according to equation (24), for each firm to choose the same threshold, which is $\overline{V}$, $\gamma_{s,i}$ has to be equal to I_i/I. In other words, firms agree on the timing of investment in the joint venture when each firm's stake is proportional to its investment. If the sharing rule is determined by Nash bargaining this occurs only when $\eta_i = I_i/I$ (see equations (13) and (14), and the point the dotted line intersects the dashed one in panel (b) of Figure 1). On the other hand, if firm i obtains the full rights to the project by compensating firm j (i.e., $\gamma_{s,i} = 1$), then each firm chooses to invest when the project value reaches $\overline{V}$ only if $C_i = I_j$. This means that firm i pays firm j the investment amount the latter contributes to the project. If the firms play a Nash bargaining game, this sharing rule is obtained only when $\eta_i = 1$ (see equations (15) and (16), and the point the solid line intersect the dashed one in panel (b) of Figure 1).

In the context of the JV scenario the following corollary follows from Propositions 1, 2, and 4.

Corollary 1 *Firms i and j agree on the exercise of their joint real option to form a joint venture in the following cases:*

1. *When the exercise decision is made first.*

2. *When the sharing rule is decided first through Nash Bargaining and:*

   (a) a combination of a stake in the project and cash transfers is used.

   (b) no cash transfer is used and $\eta_i = I_i/I$.

   (c) Firm i obtains full rights to the project and $\eta_i = 1$.

3. *When the sharing rule is decided first and equation (24) is satisfied.*

A joint venture between firms i and j is not feasible when the sharing rule is decided first through Nash Bargaining and: (1) no cash transfer is used and $\eta_i \neq I_i/I$, (2) firm i obtains full rights to the project and $\eta_i \neq 1$. In these two cases firms cannot agree on the timing of entry.

It is worth noting that a joint venture would always be feasible if the parties could write an enforceable contract on the investment threshold. Under such a scenario, forming the joint venture is always desirable for both parties, since each party's disagreement payoff is zero. However, the investment threshold is a function of the level of the state variable, which is the value of the project. Since the project is not traded on financial markets and a verifiable price for it does not exist, it would prove very difficult, if not impossible, to verify in a court whether or not the investment took place at the agreed threshold (see also the discussion at the beginning of Section 2 and footnote **??** in particular).

3.6 Multiple firms ($N \geq 3$)

There are economic situations in which more than two firms are involved in the bargaining process. For instance, an entrepreneur can be bargaining with a number of investors for outside financing, or multiple firms can be holding a joint real option to form a joint venture. Our framework can be extended to accommodate $N \geq 3$ firms.

Proposition 5 (Multiple firms ($N \geq 3$)) *The results obtained in Propositions 1 and 2 do not change when $N \geq 3$ firms bargain over the sharing rule and one firm (firm i) makes the investment timing decision. In particular, if the exercise decision is made first, the investment threshold is:*

$$\overline{V}_{N \geq 3} = \overline{V} \tag{25}$$

If the sharing rule is determined first:

$$\overline{V}_{N \geq 3} = \begin{cases} \overline{V}_s^{(\gamma)} & C_i = 0 \\ \overline{V}_s^{(C)} & \gamma_i = 0 \\ \overline{V}_s^{(\gamma,C)} = \overline{V} & 0 < \gamma_i < 1, C_i \neq 0 \end{cases} \tag{26}$$

Proof. See Appendix A ■

If the timing of the option exercise is chosen first and the Nash bargaining follows, it is still irrelevant which of the parties chooses the investment threshold. Furthermore, all payment forms (equity stakes only, cash transfer only, and a mixture of equity stakes and cash transfers) lead to the efficient outcome. If firms first bargain about the

sharing rule and only subsequently choose the exercise threshold, the exercise policy is inefficient if no cash transfers are allowed or if one firm assumes a full control over the project and provides cash compensation to all other parties.[19] Once cash transfers are allowed at least between the firm choosing the investment threshold and any other firm, efficiency is restored. Still, the investment policy maximizes in such a case the overall project value but not necessarily the individual values of firms' stakes in the project. Only if cash transfers among all firms are allowed, the optimal investment threshold maximizes all individual equity stake values.[20]

4   Effects of the value diversion threat

When interpreting Proposition 3 in the context of the EF scenario, we argued that the entrepreneur almost always prefers debt financing with terms fixed first. This result may seem intriguing, since in reality we observe a more diverse set of financial contracts. In particular, there is almost no scope for equity financing in the context of Proposition 3. In this section, we extend our analysis to include the possibility that firm i diverts a fraction of the project's value when investment takes place. We demonstrate that such a possibility of value diversion may rationalize equity financing.

Value diversion comes under many guises in corporate finance. In Jensen and Meckling (1976), a manager with a low equity stake can reduce firm's value by increasing his consumption of non-pecuniary benefits. In Burkart, Gromb, and Panunzi (2000), a controlling shareholder can extract private benefits at a deadweight cost, and when the control is transferred via a block sale the buyer receives the block's total value (i.e., private benefits as well as security benefits). Finally, in Bernardo, Cai, and Luo (2001), value diversion takes the form of poor managerial effort and empire building. If we consider the EF scenario in which firm i is interpreted as an entrepreneur and firm j as an investor, value diversion occurs, for example, when the entrepreneur consumes private benefits by shirking after investment instead of working hard on the project. This harms the investor, since it lowers the project's value. However, the entrepreneur can find shirking optimal, since he bears only part of the decline in the project's value,

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[19]Cash compensation can also be viewed as an employment contract. Then, it is not optimal for a lead inventor (firm i), who assumes the full business risk, to compensate cooperating inventors with fixed salaries.

[20]The Nash bargaining solution determines values of $N-1$ choice variables (γ_is). The maximization of the individual equity stake values of N firms requires further $N-1$ choice variables. Therefore, net payments of each firm can serve as these required additional choice variables (the new payment of Nth firm is just an opposite number to the sum of all the other $N-1$ payments. If, conversely, only one cash transfer is allowed, only one firm can optimize its investment threshold. Obviously, those two cases coincide when $N=2$.

but exclusively appropriates the benefits of private benefits consumption. As in standard moral hazard problem, we assume that value diversion is not verifiable by court (which follows from an earlier assumption that V is not verifiable). Therefore, the only way in which it can be avoided is by providing adequate incentives to the entrepreneur (firm i).

We introduce two new parameters in order to investigate the effects of value diversion in our setting. ϕ is the fraction of value firm i can divert when the investment takes place, and θ is the fraction of diverted value firm i can appropriate, whereby $0 \leq \theta \leq 1$ and $0 \leq \phi \leq 1$. In other words, value diversion is inefficient such that it has a deadweight cost of $(1 - \theta)\phi V$. Firm i has a choice between behaving (no value diversion) and misbehaving.[21] Its choice is determined by an incentive compatibility constraint (ICC). For instance, when firms i and j share the project value with no cash transfers, the ICC is $\gamma_i V \geq \gamma_i (1 - \phi)V + \theta\phi V$, which simplifies to $\gamma_i \geq \theta$.[22] In other words, firm i prefers to behave when its equity stake is at least as large as its efficiency of value diversion.

The fact that firm i has incentives to divert value when its equity stake is low affects the bargaining that takes place between the firms. To illustrate this point, we consider the case when the sharing rule is decided first. At the end of bargaining if the ICC holds (i.e., $\gamma_i \geq \theta$), firm i behaves and chooses the investment threshold that maximizes its NPV:

$$\max_V (\gamma_i V - I_i) \left(\frac{V_0}{V} \right)^\beta \quad \theta \leq \gamma_i \leq 1 \tag{27}$$

However, if the ICC is violated (i.e., $\gamma_i < \theta$), firm i diverts value. It appropriates fraction θ of the diverted value ϕV (for example to consume private benefits) and also shares the remaining value $(1 - \phi)V$ with firm j according to the sharing rule γ_i. Importantly, firm i cares about $\theta\phi V$ as well as $\gamma_i(1-\phi)V$ when choosing the investment threshold:

$$\max_V ((\gamma_i(1 - \phi) + \theta\phi)V - I_i) \left(\frac{V_0}{V} \right)^\beta \quad 0 \leq \gamma_i < \theta \tag{28}$$

As a result, when bargaining over γ_i, firms have to take into account firm i's ex post behavior as well as its investment timing decision. Thus, both the terms and timing of

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[21]One can also interpret V as the value of the project if party i (manager) exerts effort and $V(1-\theta)$ otherwise. Effort is not contractible so the moral hazard problem occurs. Party i will only exert effort if its compensation is greater than the cost of effort equal to $\theta\phi V$. A related problem with the benefit of effort being stochastic is analyzed in Grenadier and Wang (2005).

[22]This is also true when cash transfers are allowed. On the other hand, when firm i obtains full rights to the project by transferring C_i to firm j, the ICC is: $V - C_i \geq (1 - \phi)V + \theta\phi V - C_i$, which simplifies into: $1 \geq \theta$. In other words, firm i always behaves in this case, since it has a full claim on the residual project value.

investment depend on θ and ϕ.

Proposition 6 (Value diversion) *If firms i and j decide the sharing rule first without cash transfers and if firm i can divert value, the terms and timing of investment are as follows.*

$$(\gamma_{v,i}, \overline{V}_v) = \begin{cases} (\gamma_{s,i}, \overline{V}_s^{(\gamma)}) & 0 \le \theta \le \underline{\theta}, \quad 0 \le \phi \le 1 \\[2mm] \left(\gamma_{s,i} - \dfrac{\theta\phi(1-\gamma_{s,i})}{1-\phi}, \dfrac{\overline{V}_s^{(\gamma)}}{1-\phi+\theta\phi}\right) & \phi \le \phi^*, \quad \underline{\theta} < \theta \le \overline{\theta}, \quad 0 \le \phi \le \underline{\phi} \\[2mm] (\theta, kI_i/\theta) & \phi > \phi^*, \quad \underline{\theta} < \theta \le \overline{\theta}, \quad 0 \le \phi \le \underline{\phi} \\[2mm] (\theta, kI_i/\theta) & \theta \le \theta^*, \quad \underline{\theta} < \theta \le \overline{\theta}, \quad \underline{\phi} < \phi \le 1 \\[2mm] (0, kI_i/\theta\phi) & \theta > \theta^*, \quad \underline{\theta} < \theta \le \overline{\theta}, \quad \underline{\phi} < \phi \le 1 \\[2mm] \left(\gamma_{s,i} - \dfrac{\theta\phi(1-\gamma_{s,i})}{1-\phi}, \dfrac{\overline{V}_s^{(\gamma)}}{1-\phi+\theta\phi}\right) & \overline{\theta} < \theta \le 1, \quad 0 \le \phi \le \underline{\phi} \\[2mm] (0, kI_i/\theta\phi) & \overline{\theta} < \theta \le 1, \quad \underline{\phi} < \phi \le \overline{\phi} \\[2mm] \emptyset & \overline{\theta} < \theta \le 1, \quad \overline{\phi} < \phi \le 1 \end{cases}$$

where $\underline{\theta}, \overline{\theta}, \underline{\phi}, \overline{\phi}, \phi^*$ and θ^* are defined by equations A.3, A.4, A.5, A.6, A.7, and A.8, respectively, in Appendix A.

Proof. See Appendix A ■

[Insert Figure 2 about here]

From Proposition 2, we know that when firms bargain over the sharing rule with no cash transfers, the outcome is $\gamma_{s,i}$ as given in equation (13). For low values of θ ($\theta \le \underline{\theta}$), this outcome stays the same, since $\gamma_{s,i} \ge \theta$. Firm i diverts value very inefficiently, so it has no incentives to misbehave. This outcome corresponds to region *No diversion threat* in Figure 2 (the set of parameter values used is $\eta_i = 0.1$, $\beta = 2$ and $I_i = I_j$). However when θ is not low ($\theta > \underline{\theta} = 0.367$), $\gamma_{s,i} < \theta$ and firm i has incentives to misbehave. In this situation, two things can happen. Firms can either agree to *raise* firm i's equity stake to θ in order to incentivize it to behave, or *lower* it to $\gamma_{s,i} - \theta\phi(1-\gamma_{s,i})/(1-\phi)$ in order to take into account the fact that firm i will divert value when investment occurs.

The former outcome becomes more likely when θ gets lower and ϕ gets higher (see *Incentivization* in Figure 2). When θ gets lower, a smaller increase in firm i's equity stake is sufficient to incentivize this firm. When ϕ gets higher, it is more costly to let firm i misbehave because the value lost due to its misbehavior $(1 - \theta)\phi V$ becomes larger. When incentivization occurs, the value of firm i's option to invest increases (see $(\theta = 0.5, \phi = 0.4)$ versus $(\theta < 0.367)$ in Figure 3) due to the fact its equity stake jumps from $\gamma_{s,i}$ to θ. It follows that firm i accelerates the investment.

[Insert Figure 3 about here]

The latter outcome becomes more likely when θ gets higher and ϕ gets lower (see *Diversion with reduced stake* in Figure 2). When θ gets higher, incentivizing firm i leaves only a small fraction of the project value $1 - \theta$ for firm j. When ϕ gets lower, it becomes less costly to let firm i misbehave, since it only diverts a small fraction of the project's value. For both of these reasons, firm j prefers to let firm i misbehave rather than relinquishing part of its equity in order to incentivize firm i. When diversion occurs, the value of firm i's option to invest falls (see $(\theta = 0.7, \phi = 0.4)$ versus $(\theta < 0.367)$ in Figure 3) due to the deadweight cost of value diversion. Thus, firm i delays the investment.

When firm i is let to misbehave, its stake in the project goes down as ϕ gets larger. This is because firm j anticipates that a larger fraction of the project value will be diverted and bargains firm i's stake down. However, firm i's stake cannot go below zero due to limited liability. Consequently, when ϕ exceeds $\underline{\phi}(\theta)$, firm j receives 100% of the project's value $(1 - \phi)V$, and firm i is compensated by the value it appropriates from the diverted value $\theta\phi V$ (*Pure diversion* in Figure 2). Firm i's option to invest is more valuable in the case of pure diversion compared to the case of diversion with reduced stake (see $(\theta = 0.7, \phi = 0.6)$ versus $(\theta = 0.7, \phi = 0.4)$ in Figure 3), since the limited liability prevents firm i's equity stake from becoming negative.

Interestingly, when θ and ϕ are both high ($\theta > \overline{\theta} = 0.667$ and $\phi > \overline{\phi}(\theta)$), the bargaining breaks down (*No equilibrium* region in Figure 2). The reason is as follows. One one hand, if firm j incentivizes firm i, there is too little equity left for firm j to break even: $(1 - \theta)(kI_i/\theta) < I_j$. On the other hand, if firm j lets firm i misbehave, it receives 100% of the project's value, but it again cannot break even because firm i diverts a very large fraction of the project's value: $(1 - \phi)(kI_i/\theta\phi) < I_j$. As a result, firm j refuses to contribute to the project. An increase in I_i or any action that restrains θ and ϕ, such as monitoring, can resolve the issue.

22

Furthermore, it is worth pointing out that $\partial\bar{\theta}/\partial k > 0$ and $\partial\bar{\phi}/\partial k > 0$ so the *No equilibrium* region shrinks when project volatility increases. This result is fairly intuitive as higher volatility boosts the option value to invest in the project and foregoing the investment due to the lack of agreement becomes more costly. Finally, as $\partial\underline{\theta}/\partial k \gtrless 0$ for $\eta_i \gtrless I_i/I$, the range of parameter values for which the threat of diversion exists shrinks with volatility as long as the bargaining power of the firm that can make such a threat is sufficiently high. Again, the latter result is a consequence of firm i receiving a higher equity stake, which makes value diversion less attractive.

In section 3.3 we discussed that firm i almost always prefers to obtain full rights to the project by compensating firm j with a cash transfer at the time of investment (see Proposition 3). In terms of the EF scenario, this meant that the entrepreneur's predominant choice is pure debt financing and there is no demand for pure equity financing. Now, we allow for the possibility that firm i can divert value at the time of investment and re-examine firm i's choice between obtaining full rights to the project ($\gamma_{s,i} = 1$) and sharing the project's value without cash transfers ($C_{s,i} = 0$).

Proposition 7 (Firm i's choice when it can misbehave) *Results obtained in Proposition 3 continue to hold except in the following two cases when firm i prefers sharing the project's value without cash transfers:*

1. *When firm i's stake is $\gamma_{v,i} = \theta$ due to incentive compatibility and $\eta_i < \eta^*$, where:*

$$\eta^* = \beta - \frac{I_i}{I}\frac{\beta - 1}{\theta^k} \tag{29}$$

2. *When firm i's stake is $\gamma_{v,i} = 0$ due to limited liability and $\eta_i < \eta^{**}$, where:*

$$\eta^{**} = \beta - \frac{I_i}{I}\frac{\beta - 1}{(\theta\phi)^k} \tag{30}$$

Proof. See Appendix A ∎

In regards to the EF scenario, Proposition 7 implies that pure debt financing is no longer the entrepreneur's dominant financing choice, such that he can demand pure equity financing as well. The fact that the entrepreneur can prefer pure equity financing when either the incentive compatibility constraint or the limited liability constraint is binding makes intuitive sense. In the former case, his equity stake is raised to θ, so that he behaves. In the latter case, it is raised to zero, so that limited liability is not violated. Of course, in both cases, the increase in his equity stake boosts his expected NPV and makes equity financing more attractive. Furthermore, the increase in his

equity stake is particularly valuable for the entrepreneur when his bargaining power η_i is low, since when η_i is high he is already appropriating a large fraction of the surplus.

In summary, the findings in this section provide the following insights. The presence of value diversion threat has an impact on both the exercise and the sharing rule decisions and causes distortions in investment timing. The impact of value diversion threat on firm i's value of the option to invest is illustrated in Figure 3. Furthermore, value diversion threat rationalizes pure equity contracts when firm i is incentivized to behave. This makes intuitive sense, since firm i's equity stake in the project is raised for the sake of incentivization and as a result a pure equity contract becomes more attractive for this firm. Finally, the possibility of value diversion leads to a rich set of interesting scenarios as illustrated in Figure 2. For example, when the fraction of value diverted is small (i.e., the parameter ϕ is low) and the deadweight cost of misbehavior is low (i.e., the parameter θ is high), firms find it optimal to let firm i misbehave rather than incentivize it.[23]

5 Conclusions

We study the optimal exercise of jointly held real options in a two-stage framework, in which firms have to agree on the terms and timing of investment. Firms can have different levels of bargaining power and each bears a specific portion of the investment cost. They decide on either the timing or the terms first. Furthermore, we allow for the diversion of some of the project's value by the firm that makes the timing decision.

In this framework, we obtain a broad set of results on investment efficiency and financing policy. First, when the exercise decision is made first, investment always occurs at a socially optimal time regardless of the financing policy and which firm makes the exercise decision. However, when the sharing rule is determined first, the first-best solution can be attained only if a combination of a stake in the project and cash transfers is used. This finding can explain the extensive use of participating convertible preferred equity in venture capital contracting, which can be viewed as a combination of straight preferred stock (a debt-like instrument) and common stock (Kaplan and Strömberg (2003)).

One of the interpretations of our framework is the interaction of a wealth-constrained entrepreneur with an investor. When the sharing rule is determined first, a debt contract that resembles a loan commitment is the dominant form of financing for the entrepreneur. This result may help explain the high demand for loan commitment

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[23]This case can be reinterpreted as a combination of a relatively high cost and low benefit of effort, which results in no effort being exerted in the equilibrium.

contracts, which constitute over 80% of all commercial and industrial loans in the US (Shockley and Thakor (1997)).

Equity financing is rationalized when there is a possibility that the entrepreneur can divert value from the project at a deadweight cost. Equity financing becomes more attractive for instance when the investor incentivizes the entrepreneur by increasing his equity stake. New ventures in high-tech industries are more prone to imposing a value diversion threat, since such firms can divert value more easily and efficiently owing to the higher difficulty in verifying their cash flows compared to, for instance, mature firms in traditional industries. Therefore, equity financing should be particularly attractive for such firms, a conjecture that agrees with the results in Berger and Udell (1998).

The framework developed in the paper can potentially be extended further by relaxing some of the model's limiting assumptions. For instance, more than one party can be allowed to divert value from the project to investigate the impact of multiple value diversion threats. Furthermore, a more extensive analysis of the moral hazard problem could follow from our framework. Currently, we only allow for a deterministic technology of value diversion (equivalent to deterministic benefit of effort). Introducing randomness is likely to reduce the range of parameter values for which incentivizing the firm that takes the investment decision is optimal. Allowing for a broader contract space could, in turn, enable a closer alignment of interests and increase the region in which incentivization prevails. Finally, it would be interesting to relax the assumption of risk neutrality. In a setting related to ours, Cvitanić et al (2011) demonstrate that risk aversion affects the design of venture capital contracts, whereas Hugonnier and Morellec (2007) show that a risk averse entrepreneur speeds up investment and invests inefficiently early. At this stage, we are able to make the following preliminary conjectures concerning the effect of risk aversion on the results. First, if firm i is risk averse and allowed to divert value, firm j anticipates that the former will accelerate investment. Furthermore, firm j also anticipates that the investment will be accelerated further if it incentivizes firm i, since the latter's equity stake is increased. Thus, we conjecture that the set of parameter values for which firm i is incentivized will be smaller to reflect the inefficient acceleration in investment resulting from risk aversion. Second, when firm j lets firm i misbehave, investment is typically excessively delayed due to the deadweight cost of value diversion that lowers the project value. However, a risk-averse firm i will not delay investment as much, which works in favor of firm j. Thus, we expect that the region in which firm j lets firm i misbehave will expand. We leave a formal analysis of these extended decision frameworks for future research.

A Proofs of Propositions

Proof of Proposition 1. The first order conditions of (5) yield the sharing rule γ_i and C_i as a function of the investment threshold V:

$$
\begin{aligned}
\gamma_i(V) &= \frac{I_i + C_i + \eta_i(V - I)}{V} \\
C_i(V) &= \gamma_i V - I_i - \eta_i(V - I)
\end{aligned}
$$

Plugging $\gamma_i(V)$ and $C_i(V)$ into (6) yields:

$$
\max_V \left[\eta_i(V - I)\right] \left(\frac{V_0}{V}\right)^\beta
$$

The solution to this maximization problem is $\overline{V}_e = kI = \overline{V}$.

For the polar case when there is no cash exchange ($C_i = C_j = 0$):

$$
\gamma_{e,i} = \frac{I_i + C_i + \eta_i(\overline{V}_e - I)}{\overline{V}_e} = \frac{I_i + \eta_i(k - 1)I}{kI}
$$

For the other polar case when firm i gains the rights to the project ($\gamma_i = 1$, $\gamma_j = 0$):

$$
C_{e,i} = \gamma_i \overline{V}_e - I_i - \eta_i(\overline{V}_e - I) = I_j + \eta_j(k - 1)I
$$

For the remaining cases, any $(\gamma_{e,i}, C_{e,i})$ pair is feasible if it satisfies:[24]

$$
\gamma_{e,i} kI + C_{e,i} - I_i = \eta_i(k - 1)I
$$

∎

Proof of Proposition 2. The first order condition of (11) yields the investment threshold V as a function of the sharing rule γ_i and C_i:

$$
V(\gamma_i, C_i) = \frac{k(I_i + C_i)}{\gamma_i}
$$

Plugging $V(\gamma_i, C_i)$ into (12) yields:

$$
\max_{\gamma_i, C_i} \frac{[(k - 1)(I_i + C_i)]^{\eta_i}}{\left[\frac{k(I_i + C_i)}{\gamma_i} - (k - 1)(I_i + C_i) - I\right]^{\eta_i - 1}} \left(\frac{\gamma_i V_0}{k(I_i + C_i)}\right)^\beta
$$

---

[24]This condition simply says that any $(\gamma_{e,i}, C_{e,i})$ pair is feasible so long as firm i's net claim $\gamma_{e,i}\overline{V} + C_{e,i} - I_i$ is worth fraction η_i of the project's surplus $\overline{V} - I$.

We maximize the natural logarithm of the expression above. The first-order conditions yield:

$$\gamma_i = \frac{I_i + C_i + \eta_i(k-1)(I_i + C_i)}{I + (k-1)(I_i + C_i)} \tag{A.1}$$

$$C_i = \gamma_i I \frac{\beta - \eta_i}{\beta - \gamma_i} - I_i \tag{A.2}$$

Solving (A.1) and (A.2) yields $\gamma_{s,i} = \eta_i$ and $C_{s,i} = \eta_i I - I_i$. $V(\eta_i, \eta_i I - I_i)$ is equal to $\overline{V}_s^{(\gamma,C)} = kI = \overline{V}$.

For the polar case when there is no cash exchange, set $C_i = 0$ and use (A.1) to obtain:

$$\gamma_{s,i} = \frac{I_i + \eta_i(k-1)I_i}{I + (k-1)I_i}$$

$V([I_i + \eta_i(k-1)I_i]/[I + (k-1)I_i], 0)$ is equal to:

$$\overline{V}_s^{(\gamma)} = \frac{1 + (I_i/I)(k-1)}{1 + \eta_i(k-1)}\overline{V}$$

For the other polar case, set $\gamma_i = 1$ and use (A.2) to obtain:

$$C_{s,i} = I_j + \eta_j(k-1)I$$

$V(1, I_j + (1 - \eta_i)(k-1)I)$ is equal to:

$$\overline{V}_s^{(C)} = (1 + (1 - \eta_i)(k-1))\overline{V}$$

■

Proof of Proposition 3. It is straightforward to prove the part that $NPV_{e,i} = NPV_{s,i}^{(\gamma,C)}$. Proposition 1 shows that when the exercise decision is made first, firm i always chooses the first-best investment threshold $\overline{V}$. Its net payoff at the time of investment is $\gamma_{e,i}\overline{V} - I_i$ when $C_{e,i} = 0$, $\overline{V} - C_{e,i} - I_i$ when $\gamma_{e,i} = 1$, and $\gamma_{e,i}\overline{V} - C_{e,i} - I_i$ when $0 < \gamma_{e,i} < 1$ and $C_{e,i} \neq 0$). The net payoff is equal to $\eta_i(\overline{V} - I)$ in each of these three cases. Furthermore, Proposition 2 shows that when the sharing rule is determined first, firm i still chooses $\overline{V}$ when ($0 < \gamma_{s,i} < 1$ and $C_{s,i} \neq 0$). Its net payoff in this case is $\gamma_{s,i}\overline{V} - C_{s,i} - I_i$, which is again equal to $\eta_i(\overline{V} - I)$. Consequently, we have $NPV_{e,i} = NPV_{s,i}^{(\gamma,C)}$.

Next, we prove the part that firm i always prefers to buy out firm j's ownership stake

$(\gamma_{s,i} = 1)$ rather than sharing the project's ownership with no cash transfers $(C_{s,i} = 0)$:

$$NPV_{s,i}^{(C)} = (\overline{V}_s^{(C)} - C_{s,i} - I_i)\left(\frac{V_0}{\overline{V}_s^{(C)}}\right)^\beta \lessgtr NPV_{s,i}^{(\gamma)} = (\gamma_{s,i}\overline{V}_s^{(\gamma)} - I_i)\left(\frac{V_0}{\overline{V}_s^{(\gamma)}}\right)^\beta$$

$$(1 + (1 - \eta_i)(k - 1))^{1-\beta}(k - 1)I\left(\frac{V_0}{kI}\right)^\beta \lessgtr (k - 1)I_i\left(\frac{(1 + \eta_i(k - 1))V_0}{kI + k(k - 1)I_i}\right)^\beta$$

$$(1 + (1 - \eta_i)(k - 1))^{1-\beta} \lessgtr \frac{I_i}{I}\left(\frac{(1 + \eta_i(k - 1))}{1 + (k - 1)I_i/I}\right)^\beta$$

$$\frac{1 + (1 - \eta_i)(k - 1)}{(\beta + \eta_i(1 - \eta_i)(k - 1))^\beta} \lessgtr \frac{I_i/I}{(\beta - 1 + I_i/I)^\beta}$$

Firm i always prefers the option $\gamma_{s,i} = 1$, since:[25]

$$\frac{1 + (1 - \eta_i)(k - 1)}{(\beta + \eta_i(1 - \eta_i)(k - 1))^\beta} > \frac{I_i/I}{(\beta - 1 + I_i/I)^\beta} \quad \text{for } 0 \le \eta_i \le 1$$

Finally, we prove that firm i prefers to buy out firm j's ownership stake $(\gamma_{s,i} = 1)$ rather than sharing the project's ownership with a cash transfer $(0 < \gamma_{s,i} < 1$ and $C_{s,i} \ne 0)$ when $\eta_i < 1$ and that it is indifferent between these two options when $\eta_i = 1$:

$$NPV_{s,i}^{(C)} = (\overline{V}_s^{(C)} - C_{s,i} - I_i)\left(\frac{V_0}{\overline{V}_s^{(C)}}\right)^\beta \lessgtr NPV_{s,i}^{(\gamma,C)} = (\gamma_{s,i}^{(\gamma,C)}\overline{V}_s^{(\gamma,C)} - C_{s,i}^{(\gamma,C)} - I_i)\left(\frac{V_0}{\overline{V}_s^{(\gamma,C)}}\right)^\beta$$

$$(1 + (1 - \eta_i)(k - 1))^{1-\beta}(k - 1)I\left(\frac{V_0}{kI}\right)^\beta \lessgtr \eta_i(k - 1)I\left(\frac{V_0}{kI}\right)^\beta$$

$$\left(\frac{\beta - 1}{\beta - \eta_i}\right)^{\beta-1} \lessgtr \eta_i$$

Firm i prefers the option $\gamma_{s,i} = 1$ when $\eta_i < 1$, since:

$$\left(\frac{\beta - 1}{\beta - \eta_i}\right)^{\beta-1} > \eta_i \quad \text{for } 0 \le \eta_i < 1$$

It is indifferent between the options $\gamma_{s,i} = 1$ and $0 < \gamma_{s,i} < 1$ and $C_{s,i} \ne 0$ when $\eta_i = 1$, since:

$$\left(\frac{\beta - 1}{\beta - \eta_i}\right)^{\beta-1} = \eta_i \quad \text{for } \eta_i = 1$$

∎

---

[25]In order to see why the inequality holds, set $I_i = I$ and $\eta_i = 1$, in which case the inequality becomes an equality, since both sides are equal to $1/\beta^\beta$. If η_i changes to an arbitrary value $e \in [0, 1)$, the right hand side is unaffected, but the left hand side increases, and the inequality holds. Furthermore, if I_i changes to an arbitrary value in $[0, I]$ while η_i is fixed at e, the inequality continues to hold, since the left hand side is unaffected and the right hand side is either unaffected (if $I_i = I$) or falls (if $I_i < I$). Therefore, the inequality holds for $0 \le \eta_i < 1$ and $0 \le I_i \le I$ and for $\eta_i = 1$ and $0 \le I_i < I$. Note that the case $I_i = I$ is not interesting, since firm i can finance the project without firm j. Thus, we assume $I_i < I$, in which case the inequality holds for $0 \le \eta_i \le 1$.

Proof of Proposition 4.   As shown in the proof of Proposition 2, firm i's exercise decision given a sharing rule is:

$$V(\gamma_i, C_i) = \frac{k(I_i + C_i)}{\gamma_i}$$

Then, firm j's exercise decision given a sharing rule is:

$$V(\gamma_j, C_j) = \frac{k(I_j + C_j)}{\gamma_j} = \frac{k(I - I_i - C_i)}{1 - \gamma_i}$$

Setting $V(\gamma_i, C_i) = V(\gamma_j, C_j)$ and simplifying yields:

$$\gamma_i = \frac{I_i + C_i}{I}$$

Any sharing rule that satisfies this equation induces each firm to invest at the same time. Furthermore, each firm always invests optimally, since: $V(\gamma_i, \gamma_i I - I_i) = \overline{V}$. ∎

Proof of Proposition 5.   We first prove that results in Proposition 1 stay the same when firm i makes the exercise decision first and then many firms bargain over the sharing rule. In this situation the bargaining problem is:

$$\max_{\gamma_i, C_i, \gamma_j, C_j, \ldots} \left[\gamma_i V - C_i - I_i\right]^{\eta_i} \left[\gamma_j V - C_j - I_j\right]^{\eta_j} \cdots$$

The first order conditions for γ_i and γ_j imply:

$$\frac{\eta_i V}{\gamma_i V - C_i - I_i} = \frac{\eta_j V}{\gamma_j V - C_j - I_j}$$

The cross multiplication yields:

$$\eta_i \gamma_j V - \eta_i C_j - \eta_i I_j = \eta_j(\gamma_i V - C_i - I_i)$$

There are $N - 1$ many such equations for firm pairs i and j ($j \neq i$). Adding them up yields:

$$\eta_i(1 - \gamma_i)V + \eta_i C_i - \eta_i(I - I_i) = (1 - \eta_i)(\gamma_i V - C_i - I_i)$$

since $\Sigma \gamma_i = 1$, $\Sigma C_i = 0$, $\Sigma \eta_i = 1$, and $\Sigma I_i = I$. Then, solving for γ_i yields the same first order condition in Proposition 1.

Similarly, the first order conditions for C_i and C_j imply:

$$\frac{-\eta_i}{\gamma_i V - C_i - I_i} = \frac{-\eta_j}{\gamma_j V - C_j - I_j}$$

Taking the cross multiplication, adding $N - 1$ such equations, and solving for C_i yields the same first order condition in Proposition 1. As a result, the rest of the proof becomes identical

29

to the proof of Proposition 1.

We next prove that results in Proposition 2 stay the same when many firms bargain over the sharing rule first and then firm i makes the exercise decision. In this situation, once the sharing rule is fixed firm i chooses $V(\gamma_i, C_i) = k(I_i + C_i)/\gamma_i$ (see the proof of Proposition 2). Firms bargain anticipating this threshold such that:

$$\max_{\gamma_i, C_i, \gamma_j, C_j, \dots} [\gamma_i \frac{k(I_i + C_i)}{\gamma_i} - C_i - I_i]^{\eta_i} [\gamma_j \frac{k(I_i + C_i)}{\gamma_i} - C_j - I_j]^{\eta_j} \cdots \left(\frac{\gamma_i V_0}{k(I_i + C_i)} \right)^{\beta}$$

The first order conditions for γ_i and γ_j imply:

$$\frac{\beta}{\gamma_i} - \frac{\gamma_j}{\gamma_i} \frac{\eta_j k(C_i + I_i)}{\gamma_j k(C_i + I_i) - \gamma_i(C_j + I_j)} - \cdots = \frac{\eta_j k(C_i + I_i)}{\gamma_j k(C_i + I_i) - \gamma_i(C_j + I_j)}$$

Define the right hand side of above equation as Z_j and note that $Z_j = Z$ for all $j \neq i$. Then the above equation becomes:

$$\frac{\beta}{\gamma_i} - \frac{(1 - \gamma_i)Z}{\gamma_i} = Z$$

which is equivalent to:

$$\beta = \frac{\eta_j k(C_i + I_i)}{\gamma_j k(C_i + I_i) - \gamma_i(C_j + I_j)}$$

The cross multiplication yields:

$$\gamma_j k(C_i + I_i) - \gamma_i(C_j + I_j) = \eta_j(k - 1)(C_i + I_i)$$

There are $N - 1$ many such equations for firm pairs i and j ($j \neq i$). Adding them up yields:

$$(1 - \gamma_i)k(C_i + I_i) - \gamma_i(-C_i + I - I_i) = (1 - \eta_i)(k - 1)(C_i + I_i)$$

since $\Sigma \gamma_i = 1$, $\Sigma C_i = 0$, $\Sigma \eta_i = 1$, and $\Sigma I_i = I$. Then, solving for γ_i yields the same first order condition in Proposition 2.

Similarly, the first order conditions for C_i and C_j imply:

$$\frac{\eta_i - \beta}{C_i + I_i} + \frac{k\gamma_j}{\gamma_i} \frac{\eta_j}{\frac{\gamma_j k(I_i + C_i)}{\gamma_i} - C_j - I_j} + \cdots = \frac{-\eta_j}{\frac{\gamma_j k(I_i + C_i)}{\gamma_i} - C_j - I_j}$$

Define the right hand side of above equation as $-Z'_j$ and note that $Z'_j = Z'$ for all $j \neq i$. Then the above equation becomes:

$$\frac{\eta_i - \beta}{C_i + I_i} + \frac{k(1 - \gamma_i)Z'}{\gamma_i} = -Z'$$

which is equivalent to:

$$C_i + I_i = \frac{\gamma_i(\beta - \eta_i)(C_j + I_j)}{k\gamma_j(\beta - \eta_i) - k(1 - \gamma_i)\eta_j - \gamma_i\eta_j}$$

Taking the cross multiplication, adding $N - 1$ such equations, and solving for C_i yields the same first order condition in Proposition 2. As a result, the rest of the proof becomes identical to the proof of Proposition 2. ∎

Proof of Proposition 6. The first order conditions of (27) and (28) yield the investment threshold V as a function of the sharing rule γ_i:

$$V(\gamma_i) = \begin{cases} kI_i/\gamma_i & \theta \leq \gamma_i \leq 1 \\ \\ kI_i/[\gamma_i(1 - \phi) + \theta\phi] & 0 \leq \gamma_i < \theta \end{cases}$$

Thus, firms i and j maximize the Nash Product N with respect to γ_i anticipating that firm i will invest when the project value reaches $V(\gamma_i)$, where:

$$N = \begin{cases} N_1 \equiv \frac{(\gamma_i V(\gamma_i) - I_i)^{\eta_i}}{((1 - \gamma_i)V(\gamma_i) - I_j)^{-\eta_j}}\left(\frac{V_0}{V(\gamma_i)}\right)^{\beta} & subject\ to\ \theta \leq \gamma_i \leq 1 \\ \\ N_2 \equiv \frac{((\gamma_i(1 - \phi) + \theta\phi)V(\gamma_i) - I_i)^{\eta_i}}{((1 - \gamma_i)(1 - \phi)V(\gamma_i) - I_j)^{-\eta_j}}\left(\frac{V_0}{V(\gamma_i)}\right)^{\beta} & subject\ to\ 0 \leq \gamma_i < \theta \end{cases}$$

Our first step is to identify the unconstrained and constrained maximums of N_1 and N_2.

The unconstrained maximum of N_1 is attained when $\gamma_{v,i} = \gamma_{s,i}$, in which case $V(\gamma_{v,i}) = \overline{V}_s^{(\gamma)}$. Incentive compatibility requires that $\gamma_{v,i}$ be at least as large as θ. Then, the maximum of N_1 is constrained when:

$$\theta > \gamma_{s,i} = \frac{I_i + \eta_i(k - 1)I_i}{I + (k - 1)I_i} \equiv \underline{\theta} \tag{A.3}$$

The binding constraint implies $\gamma_{v,i} = \theta$, in which case $V(\theta) = kI_i/\theta$. Finally, N_1 is positive so long as firm j breaks even:

$$\begin{aligned} (1 - \gamma_{v,i})V_{v,i} &\geq I_j \\ (1 - \theta)\frac{kI_i}{\theta} &\geq I_j \\ \theta &\leq \frac{I_i + (k - 1)I_i}{I + (k - 1)I_i} \equiv \overline{\theta} \end{aligned} \tag{A.4}$$

As a result, for $0 \leq \theta \leq \underline{\theta}$ firm i behaves, for $\underline{\theta} < \theta \leq \overline{\theta}$, it can be incentivized to behave, and for $\overline{\theta} < \theta \leq 1$, firms cannot agree on an equity stake that incentivizes firm i to behave.

We repeat the same analysis for N_2. The unconstrained maximum of N_2 is attained when $\gamma_{v,i} = \gamma_{s,i} - \theta\phi(1 - \gamma_{s,i})/(1 - \phi)$, in which case $V(\gamma_{v,i}) = \overline{V}_s^{(\gamma)}/(1 - \phi + \theta\phi)$. Limited liability

requires that $\gamma_{v,i}$ be at least as large as zero. Then, the maximum of N_2 is constrained when:

$$
\begin{aligned}
0 &> \gamma_{s,i} - \frac{\theta\phi(1-\gamma_{s,i})}{(1-\phi)} \\
\phi &> \frac{\theta}{\underline{\theta} + (1-\underline{\theta})\theta} \equiv \underline{\phi}
\end{aligned}
\tag{A.5}
$$

The binding constraint implies $\gamma_{v,i} = 0$, in which case $V(0) = kI_i/\theta\phi$. Finally, N_2 stays positive so long as firm j breaks even:

$$
\begin{aligned}
(1-\gamma_{v,i})(1-\phi)V_{v,i} &\geq I_j \\
(1-\phi)\frac{kI_i}{\theta\phi} &\geq I_j \\
\phi &\leq \frac{\overline{\theta}}{\overline{\theta} + (1-\overline{\theta})\theta} \equiv \overline{\phi}
\end{aligned}
\tag{A.6}
$$

As a result, for $0 \leq \phi \leq \underline{\phi}$ firm i diverts value and has some equity in the remaining project value, for $\underline{\phi} < \phi \leq \overline{\phi}$, firm i diverts value, but has no equity in the remaining project value, and for $\overline{\phi} < \phi \leq 1$, firms cannot agree on an equity stake that lets firm i misbehave.

N_1 is defined over 3 regions specified by $\underline{\theta}$ and $\overline{\theta}$. Analogously, N_2 is defined over 3 regions specified by $\underline{\phi}$ and $\overline{\phi}$. The intersection of these regions forms $3 \times 3 = 9$ regions. We label the three regions for θ as follows. If $\theta \leq \underline{\theta}$, θ is *low*; if $\underline{\theta} < \theta \leq \overline{\theta}$, θ is *moderate*; and if $\theta > \overline{\theta}$, θ is *high*. We also label the three regions for ϕ in the same fashion. Then, (low, low) refers to the intersection of $\theta \leq \underline{\theta}$ with $\phi \leq \underline{\phi}$, $(low, moderate)$ to the intersection of $\theta \leq \underline{\theta}$ with $\underline{\phi} < \phi \leq \overline{\phi}$, and so on. What remains to be done is to find $max(N_1, N_2)$ in each of these 9 regions.

For (low, low), $(low, moderate)$ and $(low, high)$, N_1 is always larger than N_2. The unconstrained maximum of N_2 is bound to be lower than the unconstrained maximum of N_1, due to the fact that the former involves a deadweight cost of value diversion. Therefore, we have:

$$
(\gamma_{v,i}, \overline{V}_v) = (\gamma_{s,i}, \overline{V}_s^{(\gamma)}) \qquad 0 \leq \theta \leq \underline{\theta},\ 0 \leq \phi \leq 1
$$

For $(moderate, low)$, firm i is either incentivized $(\gamma_{v,i} = \theta)$, else it misbehaves $(0 < \gamma_{v,i} < \theta)$. In order to establish when it is incentivized we compare:

$$
N_1(\gamma_{v,i} = \theta) \equiv \frac{\left(\theta\frac{kI_i}{\theta} - I_i\right)^{\eta_i}}{\left((1-\theta)\frac{kI_i}{\theta} - I_j\right)^{-\eta_j}} \left(\frac{V_0}{\frac{kI_i}{\theta}}\right)^{\beta}
$$

with:

$$N_2(0 < \gamma_{v,i} < \theta) \equiv \frac{\left(((\gamma_{s,i} - \frac{\theta\phi(1-\gamma_{s,i})}{1-\phi})(1-\phi) + \theta\phi)\frac{\overline{V}_s^{(\gamma)}}{1-\phi+\theta\phi} - I_i\right)^{\eta_i}}{\left((1 - (\gamma_{s,i} - \frac{\theta\phi(1-\gamma_{s,i})}{1-\phi}))(1-\phi)\frac{\overline{V}_s^{(\gamma)}}{1-\phi+\theta\phi} - I_j\right)^{-\eta_j}} \left(\frac{V_0}{\frac{\overline{V}_s^{(\gamma)}}{1-\phi+\theta\phi}}\right)^{\beta}$$

$N_1(\gamma_{v,i} = \theta)$ is equal to $N_2(0 < \gamma_{v,i} < \theta)$ when:

$$\phi^* = \frac{1}{1-\theta} - \frac{\theta\left(\frac{\frac{1-\theta}{\theta}kI_i - I_j}{\frac{1-\underline{\theta}}{\underline{\theta}}kI_i - I_j}\right)^{\frac{\beta}{\eta_j}}}{(1-\theta)\underline{\theta}} \tag{A.7}$$

such that $N_1(\gamma_{v,i} = \theta)$ is larger when $\phi > \phi^*$. Consequently, we have:

$$(\gamma_{v,i}, \overline{V}_v) = \begin{cases} \left(\gamma_{s,i} - \frac{\theta\phi(1-\gamma_{s,i})}{1-\phi}, \frac{\overline{V}_s^{(\gamma)}}{1-\phi+\theta\phi}\right) & \underline{\theta} < \theta \leq \overline{\theta}, \ 0 \leq \phi \leq \phi^* \\ (\theta, kI_i/\theta) & \underline{\theta} < \theta \leq \overline{\theta}, \ \phi^* < \phi \leq \underline{\phi} \end{cases}$$

For $(moderate, moderate)$, firm i is either incentivized $(\gamma_{v,i} = \theta)$, else it misbehaves $(\gamma_{v,i} = 0)$. In order to establish when it is incentivized we compare $N_1(\gamma_{v,i} = \theta)$ with:

$$N_2(\gamma_{v,i} = 0) \equiv \frac{\left((0(1-\phi) + \theta\phi)\frac{kI_i}{\theta\phi} - I_i\right)^{\eta_i}}{\left((1-0)(1-\phi)\frac{kI_i}{\theta\phi} - I_j\right)^{-\eta_j}} \left(\frac{V_0}{\frac{kI_i}{\theta\phi}}\right)^{\beta}$$

$N_1(\gamma_{v,i} = \theta)$ is equal to $N_2(\gamma_{v,i} = 0)$ when:

$$\theta^* = \frac{1 - (1-\phi)\phi^{\frac{\beta}{\eta_j}-1}}{1 + (1 - \phi^{\frac{\beta}{\eta_j}})\frac{1-\overline{\theta}}{\overline{\theta}}} \tag{A.8}$$

such that $N_1(\gamma_{v,i} = \theta)$ is larger when $\theta < \theta^*$. Consequently, we have:

$$(\gamma_{v,i}, \overline{V}_v) = \begin{cases} (\theta, kI_i/\theta) & \underline{\theta} < \theta \leq \theta^*, \ \underline{\phi} \leq \phi \leq \overline{\phi} \\ (0, kI_i/\theta\phi) & \theta^* < \theta \leq \overline{\theta}, \ \underline{\phi} \leq \phi \leq \overline{\phi} \end{cases}$$

For $(moderate, high)$, firm j cannot let firm i misbehave, because it cannot expect to break even if firm i misbehaves. Therefore, firm j incentivizes firm i and we have:

$$(\gamma_{v,i}, \overline{V}_v) = (\theta, kI_i/\theta) \qquad \underline{\theta} \leq \theta \leq \overline{\theta}, \ \overline{\phi} < \phi \leq 1$$

For $(high, low)$ and $(high, moderate)$, this time firm j cannot incentivize firm i because

it cannot expect to break even if firm i behaves. Therefore, firm j lets firm i misbehave and we have:

$$(\gamma_{v,i}, \overline{V}_v) = \begin{cases} \left(\gamma_{s,i} - \dfrac{\theta\phi(1 - \gamma_{s,i})}{1 - \phi}, \dfrac{\overline{V}_s^{(\gamma)}}{1 - \phi + \theta\phi} \right) & \overline{\theta} < \theta \le 1, \ 0 \le \phi \le \underline{\phi} \\ (0, kI_i/\theta\phi) & \overline{\theta} < \theta \le 1, \ \underline{\phi} < \phi \le \overline{\phi} \end{cases}$$

Finally, for $(high, high)$, N_1 and N_2 are both negative, since firm j can break even neither by incentivizing firm i nor by letting it misbehave. Consequently, the bargaining between the firms fails in this region. ∎

Proof of Proposition 7. We compare firm i's expected NPV when it receives full rights to the project $NPV_{s,i}^{(C)}$ with its expected NPV when it is offered $\gamma_{v,i} = \theta$ by firm j due to incentive compatibility $NPV_{s,i}^{(\theta)}$:

$$NPV_{s,i}^{(C)} = (\overline{V}_s^{(C)} - C_{s,i} - I_i) \left(\frac{V_0}{\overline{V}_s^{(C)}} \right)^{\beta} \lesseqgtr NPV_{s,i}^{(\theta)} = (\theta(kI_i/\theta) - I_i) \left(\frac{V_0}{(kI_i/\theta)} \right)^{\beta}$$

$$(1 + (1 - \eta_i)(k-1))^{1-\beta}(k-1)I \left(\frac{V_0}{kI} \right)^{\beta} \lesseqgtr (k-1)I_i \left(\frac{\theta V_0}{kI_i} \right)^{\beta}$$

$$(1 + (1 - \eta_i)(k-1))^{1-\beta} \lesseqgtr \frac{I_i}{I} \left(\frac{\theta I}{I_i} \right)^{\beta}$$

The right hand side of the inequality above is larger when $\eta_i < \eta^* \equiv \beta - (I_i(\beta - 1))/(I\theta^k)$, in which case firm i prefers the sharing rule with no cash transfers.

We also compare $NPV_{s,i}^{(C)}$ with firm i's expected NPV when it is offered $\gamma_{v,i} = 0$ by firm j due to limited liability $NPV_{s,i}^{(0)}$:

$$NPV_{s,i}^{(C)} = (\overline{V}_s^{(C)} - C_{s,i} - I_i) \left(\frac{V_0}{\overline{V}_s^{(C)}} \right)^{\beta} \lesseqgtr NPV_{s,i}^{(0)} = (\theta\phi(kI_i/\theta\phi) - I_i) \left(\frac{V_0}{(kI_i/\theta\phi)} \right)^{\beta}$$

$$(1 + (1 - \eta_i)(k-1))^{1-\beta}(k-1)I \left(\frac{V_0}{kI} \right)^{\beta} \lesseqgtr (k-1)I_i \left(\frac{\theta\phi V_0}{kI_i} \right)^{\beta}$$

$$(1 + (1 - \eta_i)(k-1))^{1-\beta} \lesseqgtr \frac{I_i}{I} \left(\frac{\theta\phi I}{I_i} \right)^{\beta}$$

The right hand side of the inequality above is larger when $\eta_i < \eta^{**} \equiv \beta - (I_i(\beta - 1))/(I(\theta\phi)^k)$, in which firm i prefers the sharing rule with no cash transfers. ∎

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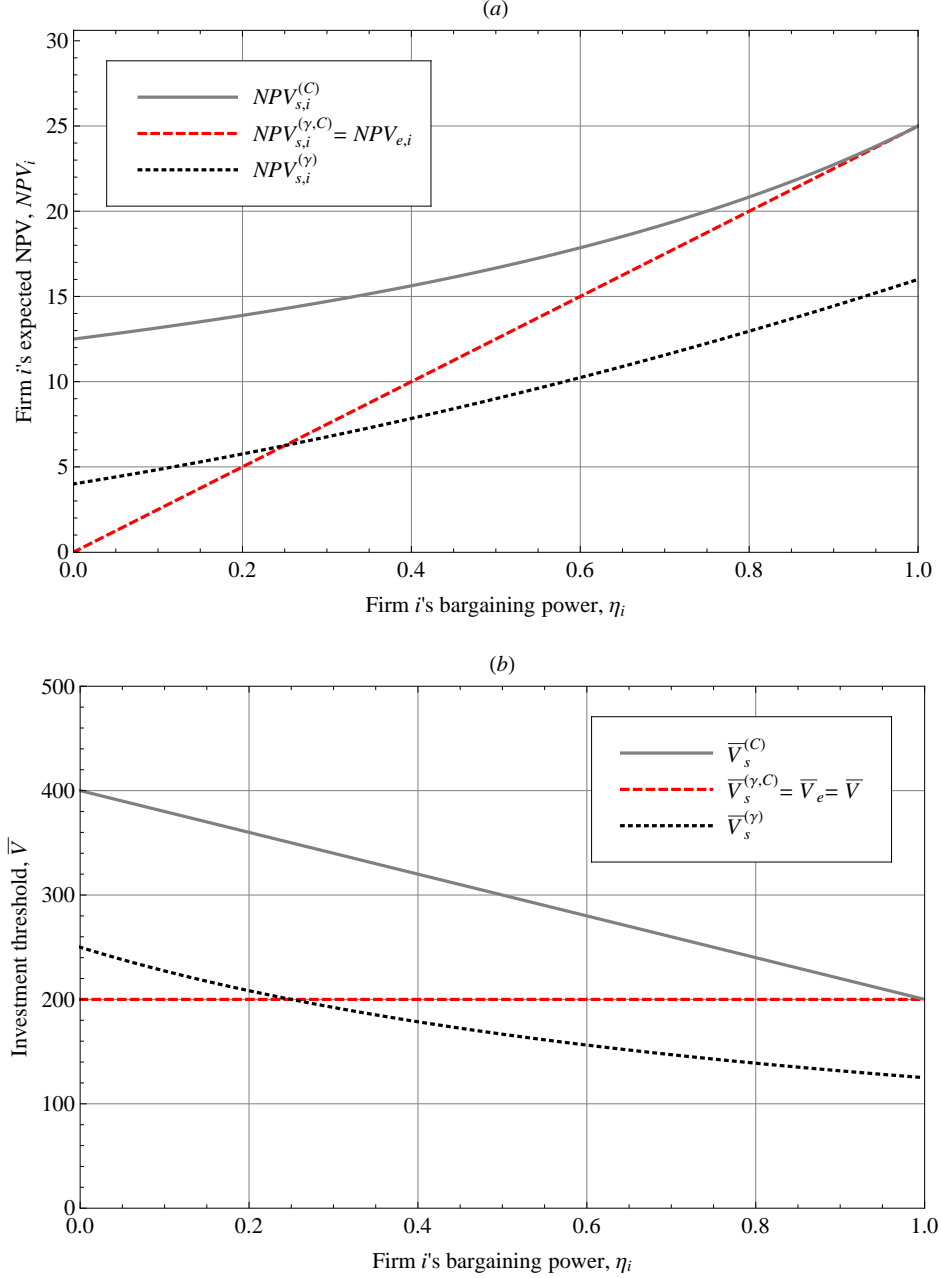
Figure 1: Firm i's payoff and investment threshold

In panel (a), firm i's expected NPV with cash transfers, $NPV_{s,i}^{(C)}$ (the solid line), with equity stake transfers, $NPV_{s,i}^{(\gamma)}$ (the dotted line), and with both cash and equity stake transfers, $NPV_{s,i}^{(\gamma,C)}$ (the dashed line), which coincides with the "exercise decision first, sharing rule second" scenario, $NPV_{e,i}$, are depicted. In panel (b), the solid and dotted lines represent the optimal investment threshold with cash transfers, $\overline{V}_s^{(C)}$, and equity stake transfers, $\overline{V}_s^{(\gamma)}$, respectively. The dashed line represents the first best-investment threshold, $\overline{V}$, which coincides with both the "exercise decision first, sharing rule second" threshold $\overline{V}_e$ as well as with the threshold in a scenario with both cash and equity stake transfers, $\overline{V}_s^{(\gamma,C)}$. The set of parameter values used in the plots is $\{I_i, I_j, V_0, \beta\} = \{25, 75, 100, 2\}$.
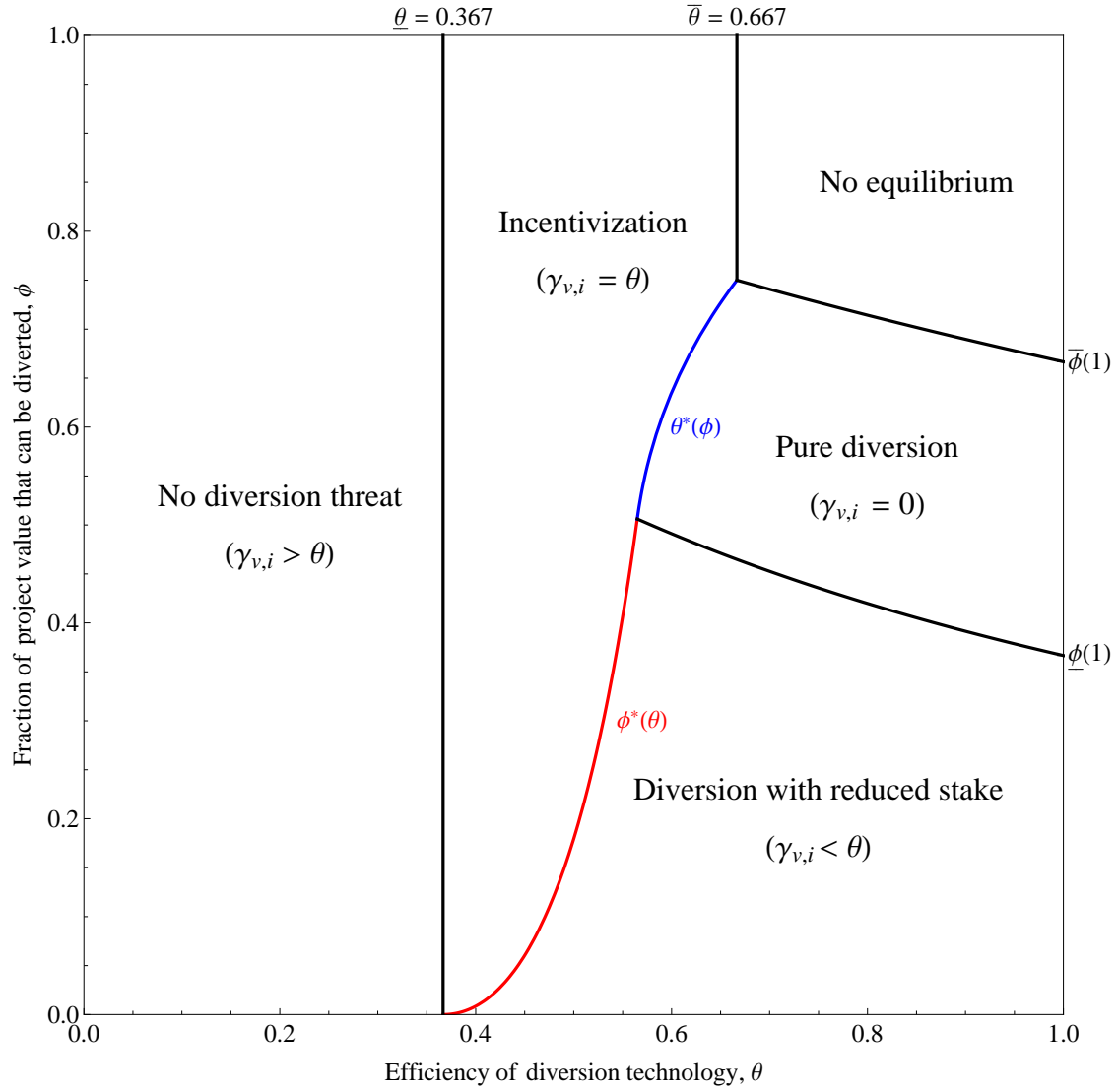
Figure 2: The impact of the value diversion threat on the sharing rule

Parameter ϕ denotes the fraction of project value firm i can divert and θ is the fraction of diverted value it can appropriate. Five regions correspond to qualitatively different outcomes of bargaining in Proposition 6. The set of parameter values used in the plot is $\{\eta_i, \beta\} = \{0.1, 2\}$ and $I_i = I_j$.
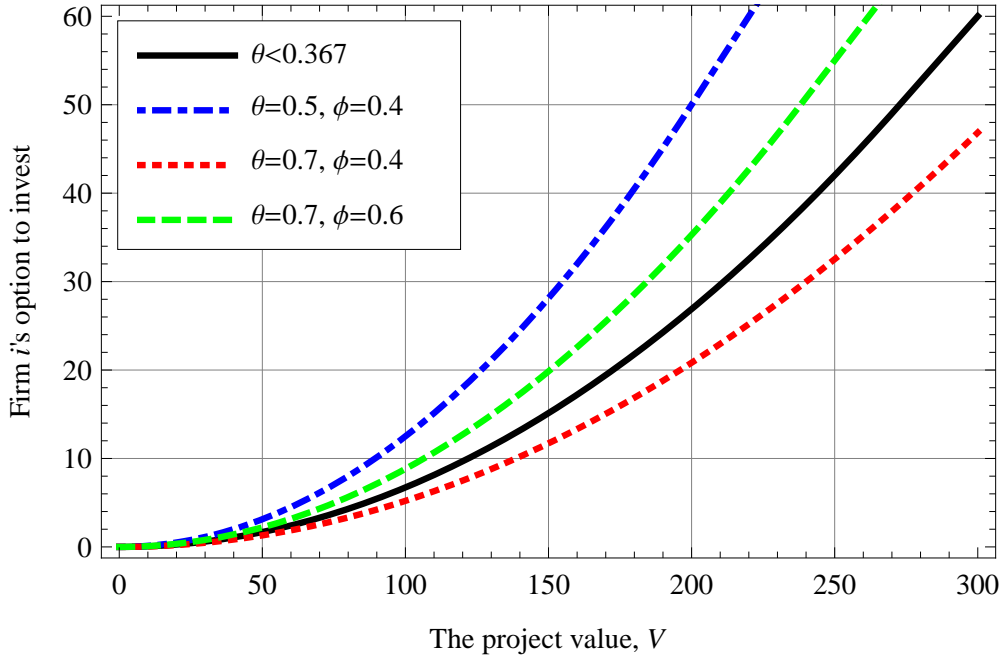
Figure 3: The The impact of value diversion threat on the value of firm i's option to invest

Parameter ϕ denotes the fraction of project value firm i can divert and θ is the fraction of diverted value it can appropriate. Solid, dash-dotted, dotted, and dashed lines correspond to the qualitatively different outcomes of bargaining in Proposition 6 where $(0 \leq \theta \leq \underline{\theta}, 0 \leq \phi \leq 1)$, $(\phi > \phi^*, \underline{\theta} < \theta \leq \overline{\theta}, 0 \leq \phi \leq \underline{\phi})$, $(\overline{\theta} < \theta \leq 1, 0 \leq \phi \leq \underline{\phi})$, and $(\overline{\theta} < \theta \leq 1, \underline{\phi} < \phi \leq \overline{\phi})$ respectively. The set of parameter values used in the plot is $\{\eta_i, \beta\} = \{0.1, 2\}$ and $I_i = I_j$.

Table 1: The project's capital and ownership structures

The project's capital and ownership structures are summarized under the scenarios EF and JV and for different levels of γ_i and C_i. Under the EF scenario, firm i is an entrepreneurial firm and firm j is an investor. Under the JV scenario, firms i and j, which may be in the same industry, enter into a joint venture. γ_i (γ_j) is the fraction of the project owned by firm i (j). C_i is a fixed payment made by firm i to firm j, such that $C_j = -C_i$.

		EF scenario	JV scenario
$\gamma_i = 1$	$C_i > 0$	pure debt	firm i buys out firm j's stake
$\gamma_i < 1$	$C_i = 0$	pure equity	shared ownership without side payments
$\gamma_i < 1$	$C_i \neq 0$	debt-equity mix	shared ownership with side payments
$\gamma_i = 0$	$C_i < 0$	cashing out	firm i sells out its stake to firm j