Questionnaire “Logic and Computability”
Summer Term 2024

Contents

11 Temporal Logic

11.1 Temporal Logic

11.1.1 Give the definition of a *Kripke structure*. Explain the components of the tuple a Kripke structure consists of. Give an example of a Kripke structure in the representation of a graph.

11.1.2 Give the definition of *paths* and *words* of Kripke structures. Give an example in which you draw a graph representing a Kripke structure, and give one possible infinite path and corresponding word.

11.1.3 What does a *computation tree* of a Kripke structure represent? Give an example in which you draw a graph representing a Kripke structure, and draw the first 3 levels of the computation tree of this Kripke structure.

11.1.4 The temporal operators describe properties that hold along a given infinite path ρ through the computation tree of a Kripke structure. Given two formulas φ and ψ describing state properties.

- Which are the properties that ρ needs to satisfy such that $\rho \models G\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models F\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models X\varphi$?
- Which are the properties that ρ needs to satisfy such that $\rho \models \varphi U \psi$?

11.1.5 Give the definition of the syntax of the computation tree logic $CTL^\star$. In particular, give the definition of state formulas and path formulas.

11.1.6 Give an intuitive explanation of the semantics of computation tree logic $CTL^\star$. Therefore, explain the semantics of the introduced path quantifiers and temporal operators with respect to the computation tree of a Kripke structure.

11.1.7 Translate the following sentences in computation tree logic $CTL^\star$.

- In every execution the system gives a grant infinitely often.
- There exists an execution in which the system sends a request finitely often.

11.1.8 Translate the following sentences in computation tree logic $CTL^\star$.

- For any execution, it always holds that whenever the robot visits region A, it visits region C within the next two steps.
- There exists an execution such that the robot visits region C within the next two steps after visiting region A.

11.1.9 Translate the following sentences in computation tree logic $CTL^\star$.

- The robot can visit region A infinitely often and region C infinitely often
- Always, the robot visits region A infinitely often and region C infinitely often.
- If the robot visits region A infinitely often, it should also visit region C finitely often.

11.1.10 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{a\}, \{b\}, \{\}, \{a\}, \{a, b\}^\omega$
- $\varphi = Xa \vee aUb$

Step	0	1	2	3	4	5	ω
a	0	1	1	0	0	1	1
b	0	0	0	1	0	0	1
Xa							
aUb							
$Xa \vee aUb$							

11.1.11 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b, c\}, \{a\}, \{a, b\}, (\{a\}, \{a, c\}, \{a, c\})^\omega$
- $\varphi = Ga \rightarrow (Fb \vee c)$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
Ga									
Fb									
$Fb \vee c$									
$Ga \rightarrow (Fb \vee c)$									

11.1.12 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b, c\}, \{a\}, \{a, b\}, (\{a\}, \{a, c\}, \{a, c\})^\omega$
- $\varphi = GFa \rightarrow (FG\neg b \wedge c)$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	0	0
c	0	0	0	1	0	0	0	1	1
GFa									
$FG\neg b$									
$FG\neg b \wedge c$									
$GFa \rightarrow (FG\neg b \wedge c)$									

11.1.13 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{\}, \{a\}, \{\}, \{a, b\}, \{a\}, \{a, b\}, (\{a\}, \{a, b\}, \{a\})^\omega$
- $\varphi = FGa \rightarrow FGb$

Step	0	1	2	3	4	5	ω		
a	0	1	0	1	1	1	1	1	1
b	0	0	0	1	0	1	0	1	0
FGa									
FGb									
$FGa \rightarrow FGb$									

11.1.14 Given the following execution word w of a Kripke structure. Evaluate the formula φ on w. Evaluate each sub-formula for any execution step using the provided table.

- $w = \{a\}, \{a\}, \{a\}, \{b, c\}, \{a\}, \{a, b\}(\{a\}, \{c\})^\omega$

- $\varphi = aUc \vee Fb$

Step	0	1	2	3	4	5	ω	
a	1	1	1	0	1	1	1	0
b	0	0	0	1	0	1	0	0
c	0	0	0	1	0	0	0	1
aUc								
Fb								
$aUc \vee Fb$								

11.1.15 Given the following Kripke structure $\mathcal{K}$. Does the initial state s_0 of $\mathcal{K}$ satisfy the following formulas?

- $\varphi_1 \coloneqq EXX(a \wedge b)$
- $\varphi_2 \coloneqq EXAX(a \wedge b)$

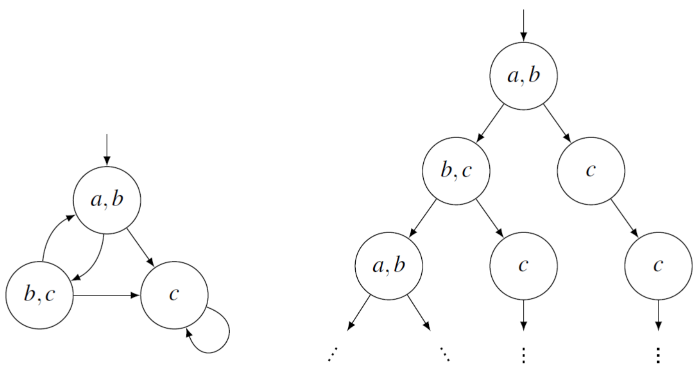

Figure 1: Left: Kripke structure of Example 7, Right: Corresponding computation tree

11.1.16 Given the following Kripke structure $\mathcal{K}$. Does the initial state s_0 of $\mathcal{K}$ satisfy the following formulas?

- $\varphi_1 \coloneqq EXp$
- $\varphi_2 \coloneqq EG\neg p$

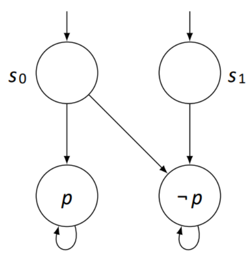

Figure 2: Kripke structure of Example 8

11.1.17 Consider an ordinary traffic junction with incoming lanes from the north, south, east and west. We want to formulate relevant constraints that a traffic light system has to fulfill.

Give a set of propositional variables that model whether the north and south *or* the east and the west get the

- green,
- yellow or
- red

light, respectively.

Formulate the following sentences using $CTL^{\star}$:

(a) The north/south lanes will never get the green light at same time as the east/west lanes.

(b) Whenever the north/south lane receive the green light it will stay green until it changes to yellow.

(c) When the east/west lane has the red light, it will eventually get the yellow and red light until the light switches to green.