# Questionnaire "Logic and Computability"
# Summer Term 2024

# Contents

# 2   SAT Solvers

## 2.1   The SAT-Problem

### 2.1.1 Define the *Boolean Satisfiability Problem*?

Solution

Given a propositional formula $\varphi$, the Boolean Satisfiability Problem asks whether there exists a model such that the formula evaluates to true.

### 2.1.2 What is the complexity of the SAT-Problem? What does its complexity imply?

Solution

The SAT-Problem is NP-complete.
Its complexity implies that it is very unlikely that there exists any polynomial algorithm.

## 2.2   The DPLL Algorithm

### 2.2.1 Given a formula $\varphi$ in CNF. (a) What is a *partial assignment* of variables? (b) What is a *total assignment* of variables? (c) What does it mean that a clause in *conflicting* with an assignment? (d) What does it mean that a clause in *satisfied* by an assignment?

Solution

*There is no solution available for this question yet.*

### 2.2.2 Let $\varphi$ be a formula in CNF and $A$ an assignment of variables in $\varphi$. Mark the following statements that are true.

☐ A clause is *satisfied* by $A$, if all variables are satisfied under $A$.

☐ If a clause is *conflicting* under an assignment $A$, if all its variables are not satisfied under $A$.

☐ If a clause is *conflicting* with an assignment $A$, all variables in the clause are given the opposite value in A.

☐ By the expression '$\varphi[A]$' we denote that a variable within $\varphi$ is satisfied under $A$.

### 2.2.3 Given the set of clauses $C_\varphi = \{\{a, \neg b\}, \{\neg a, c\}, \{b, \neg c\}, \{\neg a, \neg c\}\}$ and the assignment $A = \{\neg a\}$. Compute $\varphi[A]$.

Solution

$\varphi[A] = \{\{\neg b\}, \{b, \neg c\}, \}$

### 2.2.4 Given the set of clauses $C_\varphi = \{\{\neg a, b, c\}, \{a, c\}, \{\neg a, \neg c\}, \{\neg a, b, \neg c\}\}$ and the assignment $A = \{a\}$. Compute $\varphi[A]$.

Solution

$\varphi[A] = \{\{b, c\}, \{\neg c\}, \{b, \neg c\}\}$

2.2.5 Given the set of clauses $C_\varphi = \{\{\neg a, b\}, \{a, c\}, \{a, b, \neg c\}, \{a, \neg c\}\}$ and the assignment $A = \{\neg c\}$. Compute $\varphi[A]$.

> **Solution**
>
> $\varphi[A] = \{\{\neg a, b\}, \{a\}\}$

2.2.6 Given the set of clauses $C_\varphi = \{\{a, \neg b\}, \{\neg a, b, c\}, \{a, b, \neg c\}, \{\neg a, \neg b, \neg c\}\}$ and the assignment $A = \{b\}$. Compute $\varphi[A]$.

> **Solution**
>
> $\varphi[A] = \{\{a\}, \{\neg a, \neg c\}\}$

2.2.7 In the context of the DPLL algorithm, explain the term *decision heuristic*. Why is it important that a SAT solver implements a good decision heuristic?

> **Solution**
>
> *There is no solution available for this question yet.*

2.2.8 In the context of the DPLL algorithm, explain what a *unit clause* is. Give an example.

> **Solution**
>
> **Definition - Unit Clause.** A clause $c$ is said to be a unit clause under some assignment $A$ if the following two conditions hold:
>
> (a) The clause $c$ is not satisfied by $A$.
>
> (b) All but one of the variables in $c$ are given a value by $A$.
>
> Therefore, there is a single literal left in the set representing the clause under the assignment.
> An example would be:
>
> - $c = \{a, b, \neg c\}$
>
> - $A = \{\neg a, c\}$
>
> - $c[A] = \bot \lor b \lor \bot$, in set representation: $\{b\}$

2.2.9 In the context of the DPLL algorithm, explain what a *pure literal* is. Give an example.

> **Solution**
>
> **Definition - Pure Literal.** A literal is *pure* if its negation does not appear in the formula.
> Consider the following clauses $c_1$ and $c_2$:
>
> - $c_1 = \{a, b, \neg c\}$
>
> - $c_2 = \{\neg b, \neg c, d\}$
>
> $\neg c$ is a pure literal, since it does not appear in the positive phase.

2.2.10 In the context of the DPLL algorithm, explain why it is advantageous to apply *Boolean Constrain Propagation (BCP)* and *Pure Literals (PL)* before making a decision.

> Solution
>
> Boolean Constraint Propagation and Pure Literals are so-called heuristics. BCP and PL capture when the choices we can make are restricted in two different ways. It is advantageous to apply these heuristics before making a decision, since it reduces the amount of different assignments we have to check.

2.2.11 In the context of the DPLL algorithm, explain what *Conflict-Driven Clause Learning* is and why most modern SAT solvers use this technique.

> Solution
>
> *The idea of conflict-driven clause learning is not to repeat steps that lead to a conflict.*
>
> When executing the DPLL algorithm we can maintain a so-called conflict graph. We can use this graph to deduce the variables that lead to the conflict. In Conflict-Driven Clause Learning different SAT solvers apply different techniques to extract new *learned* clauses from this graph.
> The learned clauses help the SAT solver from repeating mistakes in different execution branches.

2.2.12 *SAT solvers* make choices based on *heuristics* on which variable and value to pick for the next decision. (a) Why is the order in which variables are chosen for decisions important for the performance of SAT solvers? (b) How does a SAT solver implementing the *dynamic largest individual sum* heuristic choose its next assignment?

> Solution
>
> *There is no solution available for this question yet.*

2.2.13 Within the context of DPLL, explain the terms *decision* and *decision level*.

> Solution
>
> *There is no solution available for this question yet.*

2.2.14 In the context of the DPLL algorithm, what does a conflict arising at decision level 0 imply about the satisfiability or unsatisfiability of a formula? Explain your answer.

> Solution
>
> *There is no solution available for this question yet.*

2.2.15 In the context of the DPLL algorithm, explain what *Boolean Constraint Propagation* is.

> Solution
>
> *There is no solution available for this question yet.*

2.2.16 Explain *conflict driven clause learning.* How do learned clauses prevent the DPLL algorithm of running into already observed conflicts multiple times?
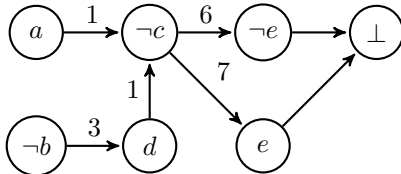
> Solution
>
> *There is no solution available for this question yet.*

2.2.17 In the context of DPLL, give the definition of the *resolution rule* used to perform a resolution proof. How can we use a resolution proof to show that an input formula is unsatisfiable?

Solution

*There is no solution available for this question yet.*

2.2.18 Consider the following conflict graph with the following set of clauses:



Clause 1: $\{\neg a, \neg c, \neg d\}$

Clause 2: $\{a, \neg d\}$

Clause 3: $\{b, d\}$

Clause 4: $\{\neg b, d, e\}$

Clause 5: $\{\neg b, \neg e\}$

Clause 6: $\{c, \neg e\}$

Clause 7: $\{c, e\}$

Give the resolution proof for the given conflict graph and clauses and state the clause to be learned from the conflict.

Solution

$$\cfrac{\cfrac{\cfrac{⑥\, c \vee \neg e \qquad ⑦\, c \vee e}{c} \qquad ①\, \neg a \vee \neg c \vee \neg d}{\neg a \vee \neg d} \qquad ③\, b \vee d}{\neg a \vee b}$$

The new learned clause is therefore Cl. 8: $\neg a \vee b$

2.2.19 Use the DPLL algorithm (*without* BCP, PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in SAT, give a satisfying model.

Clause 1: $(\neg a \vee b)$

Clause 2: $(\neg b \vee c)$

Clause 3: $(\neg c \vee d)$

Clause 4: $(\neg d \vee e)$

Clause 5: $(\neg e \vee \neg a)$

**Solution**

DPLL algorithm:

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 | 4 | 5 | 5 | 4 | 3 | 2 |
| Assignment | - | $a$ | $a,b$ | $a,b,c$ | $a,b,c,d$ | $a,b,c,d,e$ | $a,b,c,d,\neg e$ | $a,b,c,\neg d$ | $a,b,\neg c$ | $a,\neg b$ |
| Cl. 1: $\neg a, b$ | 1 | $b$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | {} ✗ |
| Cl. 2: $\neg b, c$ | 2 | 2 | $c$ | ✓ | ✓ | ✓ | ✓ | ✓ | {} ✗ | ✓ |
| Cl. 3: $\neg c, d$ | 3 | 3 | 3 | $d$ | ✓ | ✓ | ✓ | {} ✗ | ✓ | 3 |
| Cl. 4: $\neg d, e$ | 4 | 4 | 4 | 4 | $e$ | ✓ | {} ✗ | ✓ | 4 | 4 |
| Cl. 5: $\neg e, \neg a$ | 5 | $\neg e$ | $\neg e$ | $\neg e$ | $\neg e$ | {} ✗ | ✓ | $\neg e$ | $\neg e$ | $\neg e$ |
| Decision | $a$ | $b$ | $c$ | $d$ | $e$ | $\neg e$ | $\neg d$ | $\neg c$ | $\neg b$ | $\neg a$ |

| Step | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|
| Decision Level | 1 | 2 | 3 | 4 | 5 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, e$ |
| Cl. 1: $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c$ | 2 | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg c, d$ | 3 | 3 | $d$ | ✓ | ✓ |
| Cl. 4: $\neg d, e$ | 4 | 4 | 4 | $e$ | ✓ |
| Cl. 5: $\neg e, \neg a$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Decision | $b$ | $c$ | $d$ | $e$ | SAT |

Model:

$a = $ F, $b = $ T, $c = $ T, $d = $ T, $e = $ T

2.2.20 Use the DPLL algorithm with *Boolean Constrain Propagation* (*without* PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in `SAT`, give a satisfying model.

Clause 1: $(\neg a \vee b)$

Clause 2: $(\neg b \vee c)$

Clause 3: $(\neg c \vee d)$

Clause 4: $(\neg d \vee e)$

Clause 5: $(\neg e \vee \neg a)$

**Solution**

DPLL algorithm:

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $a$ | $a,b$ | $a,b,c$ | $a,b,c,d$ | $a,b,c,d,e$ | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, e$ |
| Cl. 1: $\neg a, b$ | 1 | $b$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c$ | 2 | 2 | $c$ | ✓ | ✓ | ✓ | 2 | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg c, d$ | 3 | 3 | 3 | $d$ | ✓ | ✓ | 3 | 3 | $d$ | ✓ | ✓ |
| Cl. 4: $\neg d, e$ | 4 | 4 | 4 | 4 | $e$ | ✓ | 4 | 4 | 4 | $e$ | ✓ |
| Cl. 5: $\neg e, \neg a$ | 5 | $\neg e$ | $\neg e$ | $\neg e$ | $\neg e$ | {} ✗ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | $b$ | $c$ | $d$ | $e$ | - | - | $c$ | $d$ | $e$ | ✓ |
| Decision | $a$ | - | - | - | - | $\neg a$ | $b$ | - | - | - | SAT |

Model:

$a = $ F, $b = $ T, $c = $ T, $d = $ T, $e = $ T

2.2.21 Use the DPLL algorithm with *Boolean Constrain Propagation* and *Pure Literals* (*without* clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in `SAT`, give a satisfying model.

Clause 1: $(\neg a \lor b)$

Clause 2: $(\neg b \lor c)$

Clause 3: $(\neg c \lor d)$

Clause 4: $(\neg d \lor e)$

Clause 5: $(\neg e \lor \neg a)$

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**Solution**

DPLL algorithm:

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c, \neg d$ |
| Cl. 1: $\neg a, b$ | 1 | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c$ | 2 | 2 | ✓ | ✓ | ✓ |
| Cl. 3: $\neg c, d$ | 3 | 3 | 3 | ✓ | ✓ |
| Cl. 4: $\neg d, e$ | 4 | 4 | 4 | 4 | ✓ |
| Cl. 5: $\neg e, \neg a$ | 5 | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | - |
| PL | $\neg a$ | $\neg b$ | $\neg c$ | $\neg d$ | - |
| Decision | - | - | - | - | SAT |

Model:
$a = $ F, $b = $ F, $c = $ F, $d = $ F, $e = $ F

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2.2.22 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, \neg b\}$

Clause 2: $\{a, c\}$

Clause 3: $\{b, \neg c\}$

Clause 4: $\{\neg b, d\}$

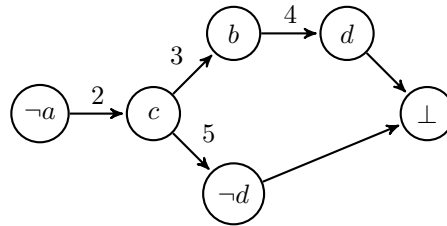Clause 5: $\{\neg c, \neg d\}$

Clause 6: $\{c, e\}$

Clause 7: $\{c, \neg e\}$

Solution

DPLL algorithm:

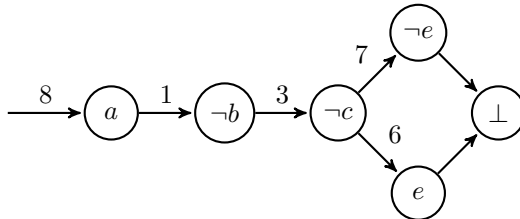| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $\neg a$ | $\neg a, c$ | $\neg a, b, c$ | $\neg a, b, c, \neg d$ | - | $a$ | $a, \neg b$ | $a, \neg b, \neg c$ | $a, \neg b, \neg c, \neg e$ |
| Cl. 1: $\neg a, \neg b$ | 1 | ✓ | ✓ | ✓ | ✓ | 1 | $\neg b$ | ✓ | ✓ | ✓ |
| Cl. 2: $a, c$ | 2 | $c$ | ✓ | ✓ | ✓ | 2 | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg c$ | 3 | 3 | $b$ | ✓ | ✓ | 3 | 3 | $\neg c$ | ✓ | ✓ |
| Cl. 4: $\neg b, d$ | 4 | 4 | 4 | $d$ | {} ✗ | 4 | 4 | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | 5 | 5 | $\neg d$ | $\neg d$ | ✓ | 5 | 5 | 5 | ✓ | ✓ |
| Cl. 6: $c, e$ | 6 | 6 | ✓ | ✓ | ✓ | 6 | 6 | 6 | $e$ | {} ✗ |
| Cl. 7: $c, \neg e$ | 7 | 7 | ✓ | ✓ | ✓ | 7 | 7 | 7 | $\neg e$ | ✓ |
| Cl. 8: $a$ | - | - | - | - | learned $a$ | 8 | ✓ | ✓ | ✓ | ✓ |
| BCP | - | $c$ | $b$ | $\neg d$ | - | $a$ | $\neg b$ | $\neg c$ | $\neg e$ | - |
| PL | - | - | - | - | - | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - | - | - | - | - | UNSAT |

Ad 5:



$$\frac{\dfrac{4.\ \neg b \vee d \qquad 5.\ \neg c \vee \neg d}{\neg b \vee \neg c} \qquad 3.\ b \vee \neg c}{\dfrac{\neg c}{a}} \qquad 2.\ a \vee c$$

Ad 10:



$$\frac{\dfrac{6.\ c \vee e \qquad 7.\ c \vee \neg e}{c} \qquad 3.\ b \vee \neg c}{\dfrac{b}{\dfrac{\neg a}{\bot}} \qquad 1.\ \neg a \vee \neg b} \qquad 8.\ a$$

**2.2.23** Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(\neg a \vee d)$

Clause 2: $(\neg d \vee c)$

Clause 3: $(\neg b \vee e)$

Clause 4: $(\neg b \vee \neg e)$

Clause 5: $(b \vee f)$

Clause 6: $(b \vee \neg f)$

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**Solution**

DPLL algorithm:

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| Assignment | - | $\neg a$ | $\neg a, c$ | $\neg a, b, c$ | $\neg a, b, c, e$ | - | $\neg b$ | $\neg b, f$ |
| Cl. 1: $\neg a, d$ | 1 | ✓ | ✓ | ✓ | ✓ | 1 | 1 | 1 |
| Cl. 2: $\neg d, c$ | 2 | 2 | ✓ | ✓ | ✓ | 2 | 2 | 2 |
| Cl. 3: $\neg b, e$ | 3 | 3 | 3 | $e$ | ✓ | 3 | ✓ | ✓ |
| Cl. 4: $\neg b, \neg e$ | 4 | 4 | 4 | $\neg e$ | {} ✗ | 4 | ✓ | ✓ |
| Cl. 5: $b, f$ | 5 | 5 | 5 | ✓ | ✓ | 5 | $f$ | ✓ |
| Cl. 6: $b, \neg f$ | 6 | 6 | 6 | ✓ | ✓ | 6 | $\neg f$ | {} ✗ |
| Cl. 7: $\neg b$ | - | - | - | - | learned $\neg b$ | 7 | ✓ | ✓ |
| BCP | - | - | - | $e$ | - | $\neg b$ | $f$ | - |
| PL | $\neg a$ | $c$ | - | - | - | - | - | - |
| Decision | - | - | $b$ | - | - | - | - | UNSAT |

Ad 5:
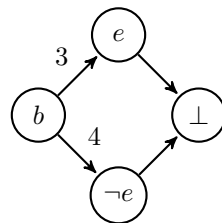


$$\frac{3.\ \neg b \vee e \qquad 4.\ \neg b \vee \neg e}{\neg b}$$

Ad 8:
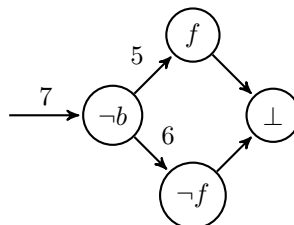


$$\frac{\dfrac{5.\ b \vee f \qquad 6.\ b \vee \neg f}{b} \qquad 7.\ \neg b}{\bot}$$

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2.2.24 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the

learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(\neg a \vee \neg c)$

Clause 2: $(b \vee c)$

Clause 3: $(\neg b \vee \neg d)$

Clause 4: $(\neg d \vee e)$

Clause 5: $(d \vee e)$

Clause 6: $(a \vee \neg c \vee \neg e)$

Clause 7: $(\neg b \vee c \vee d)$

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**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, \neg e$ | $\neg a, \neg b, c, \neg e, \neg d$ | $\neg a$ |
| Cl. 1: $\neg a, \neg c$ | $\neg a, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $b, c$ | $b, c$ | $b, c$ | $c$ | ✓ | ✓ | ✓ | $b, c$ |
| Cl. 3: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | ✓ | ✓ | ✓ | ✓ | $\neg b, \neg d$ |
| Cl. 4: $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d$ | ✓ | $\neg d, e$ |
| Cl. 5: $d, e$ | $d.e$ | $d.e$ | $d.e$ | $d.e$ | $d$ | {} ✗ | $d, e$ |
| Cl. 6: $a, \neg c, \neg e$ | $a, \neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg e$ | ✓ | ✓ | $\neg c, \neg e$ |
| Cl. 7: $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | ✓ | ✓ | ✓ | ✓ | $\neg b, c, d$ |
| Cl. 8: $a.b$ | - | - | - | - | - | $a, b$ | $b$ |
| Cl. 9: $a$ | - | - | - | - | - | - | - |
| BCP | - | - | $c$ | $\neg e$ | $\neg d$ | - | $b$ |
| PL | - | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - | - |

First Conflict in Step 6:

$$\frac{\dfrac{4.\ \neg d \vee e \qquad 5.\ d \vee e}{e} \qquad 6.\ a \vee \neg c \vee \neg e}{\dfrac{a \vee \neg c \qquad 2.\ b \vee c}{a \vee b}}$$

| Step | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| Assignment | $\neg a, b$ | $\neg a, b, \neg d$ | $\neg a, b, \neg d, c$ | $\neg a, b, \neg d, c, \neg e$ | - | $a$ | $a, \neg c$ |
| Cl. 1: $\neg a, \neg c$ | ✓ | ✓ | ✓ | ✓ | $\neg a, \neg c$ | $\neg c$ | ✓ |
| Cl. 2: $b, c$ | ✓ | ✓ | ✓ | ✓ | $b, c$ | $b, c$ | $b$ |
| Cl. 3: $\neg b, \neg d$ | $\neg d$ | ✓ | ✓ | ✓ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ |
| Cl. 4: $\neg d, e$ | $\neg d, e$ | ✓ | ✓ | ✓ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ |
| Cl. 5: $d, e$ | $d, e$ | $e$ | $e$ | {} ✗ | $d, e$ | $d, e$ | $d, e$ |
| Cl. 6: $a, \neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg e$ | ✓ | $a, \neg c, \neg e$ | ✓ | ✓ |
| Cl. 7: $\neg b, c, d$ | $d, c$ | $c$ | ✓ | ✓ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, d$ |
| Cl. 8: $a.b$ | ✓ | ✓ | ✓ | ✓ | $a, b$ | ✓ | ✓ |
| Cl. 9: $a$ | - | - | - | $a$ | $a$ | ✓ | ✓ |
| BCP | $\neg d$ | $c$ | $\neg e$ | - | $a$ | $\neg c$ | $b$ |
| PL | - | - | - | - | - | - | - |
| Decision | - | - | - | - | - | - | - |

Second Conflict in Step 11:
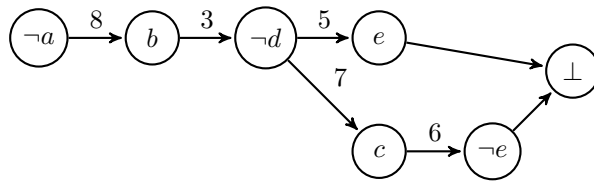


$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{6.\ a \vee \neg c \vee \neg e \qquad 7.\ \neg b \vee c \vee d}{a \vee \neg b \vee d \vee \neg e} \qquad 5.\ d \vee e}{a \vee \neg b \vee d} \qquad 3.\ \neg b \vee \neg d}{a \vee \neg b} \qquad 8.\ a \vee b}{a}}{}$$

| Step | 15 | 16 |
|---|---|---|
| Decision Level | 0 | 0 |
| Assignment | $a, \neg c, b$ | $a, \neg c, b, \neg d$ |
| Cl. 1: $\neg a, \neg c$ | ✓ | ✓ |
| Cl. 2: $b, c$ | ✓ | ✓ |
| Cl. 3: $\neg b, \neg d$ | $\neg d$ | ✓ |
| Cl. 4: $\neg d, e$ | $\neg d, e$ | ✓ |
| Cl. 5: $d, e$ | $d, e$ | $e$ |
| Cl. 6: $a, \neg c, \neg e$ | ✓ | ✓ |
| Cl. 7: $\neg b, c, d$ | $d$ | {} ✗ |
| Cl. 8: $a.b$ | ✓ | ✓ |
| Cl. 9: $a$ | ✓ | ✓ |
| BCP | $\neg d$ | - |
| PL | - | - |
| Decision | - | UNSAT |

Second Conflict in Step 16:
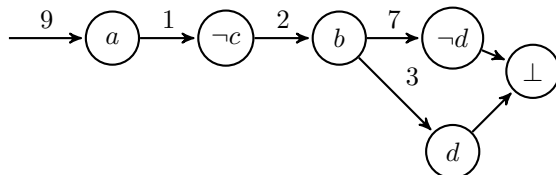


$$\cfrac{\cfrac{\cfrac{\cfrac{7.\ \neg b \vee c \vee d \qquad 3.\ \neg b \vee \neg d}{\neg b \vee c} \qquad 2.\ b \vee c}{c} \qquad 1.\ \neg a \vee \neg c}{\neg a} \qquad 9.\ a}{\bot}$$

2.2.25 Use the DPLL algorithm (*without* BCP, PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in `SAT`, give a satisfying model.

Clause 1: $(\neg a \vee b \vee \neg c)$

Clause 2: $(a \vee \neg b \vee c)$

Clause 3: $(\neg a \vee \neg b \vee c)$

Clause 4: $(a \vee b \vee \neg c)$

---

**Solution**

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 |
| Assignment | - | $a$ | $a, b$ | $a, b, c$ |
| Cl. 1: $\neg a, b, \neg c$ | $\neg a, b, \neg c$ | $b, \neg c$ | ✓ | ✓ |
| Cl. 2: $a, \neg b, c$ | $a, \neg b, c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg b, c$ | $\neg a, \neg b, c$ | $\neg b, c$ | $c$ | ✓ |
| Cl. 4: $a, b, \neg c$ | $a, b, \neg c$ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - |
| PL | - | - | - | - |
| Decision | $a$ | $b$ | $c$ | SAT |

---

2.2.26 Use the DPLL algorithm with *Boolean Constrain Propagation* (*without* PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in `SAT`, give a satisfying model.

Clause 1: $(\neg d \vee \neg b \vee \neg a)$

Clause 2: $(\neg e \vee a \vee \neg f)$

Clause 3: $(\neg a \vee c \vee b)$

Clause 4: $(f \vee a \vee e)$

Clause 5: $(d \vee \neg a \vee \neg b)$

Clause 6: $(\neg a \vee \neg c \vee b)$

**Solution**

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 |
| Assignment | - | $a$ | $a,b$ | $a,b,d$ |
| Cl. 1: $\neg d, \neg b, \neg a$ | $\neg d, \neg b, \neg a$ | $\neg d, \neg b$ | $\neg d$ | {} ✗ |
| Cl. 2: $\neg e, a, \neg f$ | $\neg e, a, \neg f$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, c, b$ | $\neg a, c, b$ | $c, b$ | ✓ | ✓ |
| Cl. 4: $f, a, e$ | $f, a, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $d, \neg a, \neg b$ | $d, \neg a, \neg b$ | $d, \neg b$ | $d$ | ✓ |
| Cl. 6: $\neg a, \neg c, b$ | $\neg a, \neg c, b$ | $\neg c, b$ | ✓ | ✓ |
| BCP | - | - | $d$ | - |
| PL | - | - | - | - |
| Decision | $a$ | $b$ | - | - |

| Step | 5 | 6 | 7 |
|---|---|---|---|
| Decision Level | 1 | 2 | 2 |
| Assignment | $a$ | $a, \neg b$ | $a, \neg b, c$ |
| Cl. 1: $\neg d, \neg b, \neg a$ | $\neg d, \neg b$ | ✓ | ✓ |
| Cl. 2: $\neg e, a, \neg f$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, c, b$ | $c, b$ | $c$ | ✓ |
| Cl. 4: $f, a, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $d, \neg a, \neg b$ | $d, \neg b$ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg c, b$ | $\neg c, b$ | $\neg c$ | {} ✗ |
| BCP | - | $c$ | - |
| PL | - | - | - |
| Decision | $\neg b$ | - | - |

| Step | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 | 4 | 5 | 5 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, e$ | $\neg a, b, c, d, e, \neg f$ |
| Cl. 1: $\neg d, \neg b, \neg a$ | $\neg d, \neg b, \neg a$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg e, a, \neg f$ | $\neg e, a, \neg f$ | $\neg e, \neg f$ | $\neg e, \neg f$ | $\neg e, \neg f$ | $\neg e, \neg f$ | $\neg f$ | ✓ |
| Cl. 3: $\neg a, c, b$ | $\neg a, c, b$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $f, a, e$ | $f, a, e$ | $f, e$ | $f, e$ | $f, e$ | $f, e$ | ✓ | ✓ |
| Cl. 5: $d, \neg a, \neg b$ | $d, \neg a, \neg b$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg c, b$ | $\neg a, \neg c, b$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | - | $\neg f$ | - |
| PL | - | - | - | - | - | - | - |
| Decision | $\neg a$ | $b$ | $c$ | $d$ | $e$ | - | SAT |

**2.2.27** Use the DPLL algorithm with *Boolean Constrain Propagation* and *Pure Literals* (*without* clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in SAT, give a satisfying model.

Clause 1: $(\neg c \lor d)$

Clause 2: $(a \lor \neg d \lor \neg e)$
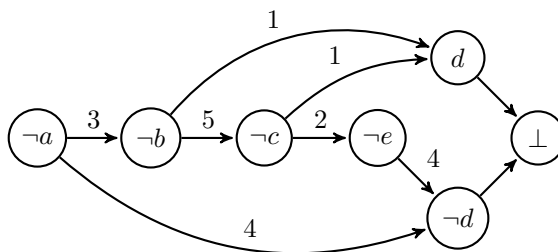
Clause 3: $(b \lor \neg c)$

Clause 4: $(c \lor e)$

Clause 5: $(\neg b \lor \neg c)$

Clause 6: $(a \lor b)$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, d$ | $a, d, e$ | $a, d, e,$ $\neg c$ |
| Cl. 1: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | ✓ | ✓ | ✓ |
| Cl. 2: $a, \neg d, \neg e$ | $a, \neg d, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | ✓ |
| Cl. 4: $c, e$ | $c, e$ | $c, e$ | $c, e$ | ✓ | ✓ |
| Cl. 5: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | ✓ |
| Cl. 6: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | - |
| PL | $a$ | $d$ | $e$ | $\neg c$ | - |
| Decision | - | - | - | - | SAT |

2.2.28 Consider the following conflict graph with the following set of clauses:



Clause 1: $\{b, c, d\}$

Clause 2: $\{c, \neg e\}$

Clause 3: $\{a, \neg b\}$

Clause 4: $\{a, \neg d, e\}$

Clause 5: $\{b, \neg c\}$

State the learned clause by making a resolution proof according to the given conflict graph and given clauses.

Solution

*There is no solution available for this question yet.*

2.2.29 Consider the formula $\varphi$ that consists of the conjunction of the following clauses:

Clause 1: $(a \vee b)$

Clause 2: $(\neg b \vee c)$

Clause 3: $(\neg a \vee \neg c)$

Clause 4: $(b \vee c)$

Clause 5: $(a \vee \neg b)$

(a) Use DPLL with learning to show that $\varphi$ is unsatisfiable. Decide variables in *alphabetic order* and starting with the *positive* phase.

(b) State and briefly explain the *resolution rule.*

(c) Using your results from 2.2.29a, give a resolution proof of the unsatisfiability of $\varphi$.

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 |
| Assignment | - | $a$ | $a, \neg c$ | $a, \neg c, b$ |
| Cl. 1: $a, b$ | $a, b$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $\neg b$ | {} ✗ |
| Cl. 3: $\neg a, \neg c$ | $\neg a, \neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 4: $b, c$ | $b, c$ | $b, c$ | $b$ | ✓ |
| Cl. 5: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ |
| BCP | - | $\neg c$ | $b$ | - |
| PL | - | - | - | - |
| Decision | $a$ | - | - | - |

Conflict in step 4



$$\frac{\dfrac{2.\ \neg b \vee c \qquad 4.\ b \vee c}{c} \qquad 3.\ \neg a \vee \neg c}{\neg a}$$

| Step | 5 | 6 | 7 |
|---|---|---|---|
| Decision Level | 0 | 0 | 0 |
| Assignment | - | $\neg a$ | $\neg a, b$ |
| Cl. 1: $a, b$ | $a, b$ | $b$ | ✓ |
| Cl. 2: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $c$ |
| Cl. 3: $\neg a, \neg c$ | $\neg a, \neg c$ | ✓ | ✓ |
| Cl. 4: $b, c$ | $b, c$ | $b, c$ | ✓ |
| Cl. 5: $a, \neg b$ | $a, \neg b$ | $\neg b$ | {} ✗ |
| Cl. 6: $\neg a$ | $\neg a$ | ✓ | ✓ |
| BCP | $\neg a$ | $b$ | - |
| PL | - | - | - |
| Decision | - | - | UNSAT |

Conflict in step 7



$$\frac{\dfrac{1.\ a \vee b \qquad 5.\ a \vee \neg b}{a} \qquad 6.\ \neg a}{\bot}$$

2.2.30 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b, \neg c\}$

Clause 2: $\{\neg b, c, d\}$

Clause 3: $\{c, d, \neg e\}$

Clause 4: $\{\neg a, d, \neg e\}$

Clause 5: $\{a, b, \neg d\}$

Clause 6: $\{c, \neg d, e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c, \neg d$ | $\neg a, \neg b, \neg c, \neg d, \neg e$ |
| Cl. 1: $a, b, \neg c$ | $a, b, \neg c$ | $b, \neg c$ | $\neg c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $c, d, \neg e$ | $c, d, \neg e$ | $c, d, \neg e$ | $c, d, \neg e$ | $d, \neg e$ | $\neg e$ | ✓ |
| Cl. 4: $\neg a, d, \neg e$ | $\neg a, d, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $a, b, \neg d$ | $a, b, \neg d$ | $b, \neg d$ | $\neg d$ | $\neg d$ | ✓ | ✓ |
| Cl. 6: $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $\neg d, e$ | ✓ | ✓ |
| BCP | - | - | $\neg c$ | $\neg d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | SAT |

2.2.31 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.
If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, \neg b\}$

Clause 2: $\{a, c, e\}$

Clause 3: $\{b, \neg d\}$

Clause 4: $\{\neg c, d, e\}$

Clause 5: $\{\neg d, e\}$

Clause 6: $\{\neg a, b\}$

Clause 7: $\{a, d, \neg e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg c$ | $\neg a, b, \neg c, e$ | $\neg a, b, \neg c, e, d$ |
| Cl. 1: $\neg a, \neg b$ | $\neg a, \neg b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, c, e$ | $a, c, e$ | $c, e$ | $c, e$ | $e$ | ✓ | ✓ |
| Cl. 3: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d, e$ | $\neg c, d, e$ | $\neg c, d, e$ | $\neg c, d, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | ✓ | ✓ |
| Cl. 6: $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a, d, \neg e$ | $a, d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d$ | ✓ |
| BCP | - | - | - | $e$ | $d$ | - |
| PL | - | $b$ | - | - | - | - |
| Decision | $\neg a$ | - | $\neg c$ | - | - | SAT |

2.2.32 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with

the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, \neg c\}$

Clause 2: $\{b, c, e\}$

Clause 3: $\{b, \neg e\}$

Clause 4: $\{\neg a, c\}$

Clause 5: $\{d, e\}$

Clause 6: $\{b, \neg d\}$

Clause 7: $\{\neg d, \neg e\}$

Clause 8: $\{a, c\}$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 |
| Assignment | - | $b$ | $b, \neg a$ | $b, \neg a, \neg c$ |
| Cl. 1: $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $\neg c$ | ✓ |
| Cl. 2: $b, c, e$ | $b, c, e$ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg e$ | $b, \neg e$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | ✓ | ✓ |
| Cl. 5: $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d, e$ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ |
| Cl. 8: $a, c$ | $a, c$ | $a, c$ | $c$ | {} ✗ |
| BCP | - | - | $\neg c$ | - |
| PL | $b$ | - | - | - |
| Decision | - | $\neg a$ | - | - |

Conflict in step 4
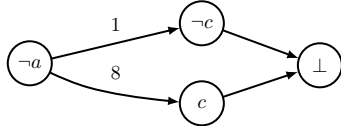


$$\frac{1.\ a \vee \neg c \qquad 8.\ a \vee c}{a}$$

| Step | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 1 | 1 |
| Assignment | $b$ | $b, a$ | $b, a, c$ | $b, a, c,$ $\neg d$ | $b, a, c,$ $\neg d, e$ |
| Cl. 1: $a, \neg c$ | $a, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $b, c, e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 5: $d, e$ | $d, e$ | $d, e$ | $d, e$ | $e$ | ✓ |
| Cl. 6: $b, \neg d$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | ✓ | ✓ |
| Cl. 8: $a, c$ | $a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | - | $e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | $\neg d$ | - | SAT |

2.2.33 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b\}$

Clause 2: $\{\neg a, c\}$

Clause 3: $\{a, \neg d\}$

Clause 4: $\{\neg b, c\}$

Clause 5: $\{\neg c, d\}$

Clause 6: $\{\neg c, e\}$

Clause 7: $\{d, \neg e\}$

### Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, \neg d$ |
| Cl. 1: $a,b$ | $a,b$ | $b$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $a, \neg d$ | $a, \neg d$ | $\neg d$ | $\neg d$ | $\neg d$ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | {} ✗ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ |
| Cl. 7: $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $\neg e$ |
| BCP | - | $b$ | $c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |

Conflict in step 5



$$\frac{\dfrac{5.\ \neg c \vee d \qquad 4.\ \neg b \vee c}{d \vee \neg b} \qquad 1.\ a \vee b}{\dfrac{d \vee a}{a} \qquad 3.\ a \vee \neg d}$$

| Step | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, d$ | $a, c, d, e$ |
| Cl. 1: $a,b$ | $a,b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $a, \neg d$ | $a, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | ✓ |
| Cl. 7: $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | ✓ | ✓ |
| Cl. 8: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | $d$ | $e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | SAT |

2.2.34 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, \neg b\}$

Clause 2: $\{a, d, e\}$

Clause 3: $\{b, \neg c\}$

Clause 4: $\{c, \neg d, e\}$

Clause 5: $\{\neg c, e\}$

Clause 6: $\{\neg a, b\}$

Clause 7: $\{a, c, \neg e\}$



| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg c$ | $\neg a, b, \neg c,$ $\neg e$ | $\neg a, b, \neg c,$ $\neg e, \neg d$ |
| Cl. 1: $\neg a, \neg b$ | $\neg a, \neg b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, d, e$ | $a, d, e$ | $d, e$ | $d, e$ | $d, e$ | $d$ | $\{\}$ ✗ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $\neg d, e$ | $\neg d$ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a, c, \neg e$ | $a, c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $\neg e$ | ✓ | ✓ |
| BCP | - | - | - | $\neg e$ | $\neg d$ | - |
| PL | - | $b$ | - | - | - | - |
| Decision | $\neg a$ | - | $\neg c$ | - | - | - |

Conflict in step 6



$$\frac{\dfrac{2.\ a \vee d \vee e \qquad 4.\ c \vee \neg d \vee e}{a \vee e \vee c} \qquad 7.\ a \vee c \vee \neg e}{a \vee c}$$

| Step | 7 | 8 | 9 |
|---|---|---|---|
| Decision Level | 1 | 1 | 1 |
| Assignment | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c,$ $e$ |
| Cl. 1: $\neg a, \neg b$ | ✓ | ✓ | ✓ |
| Cl. 2: $a, d, e$ | $d, e$ | $d, e$ | ✓ |
| Cl. 3: $b, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 4: $c, \neg d, e$ | $c, \neg d, e$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $e$ | ✓ |
| Cl. 6: $\neg a, b$ | ✓ | ✓ | ✓ |
| Cl. 7: $a, c, \neg e$ | $c, \neg e$ | ✓ | ✓ |
| Cl. 8: $a, c$ | $c$ | ✓ | ✓ |
| BCP | $c$ | $e$ | - |
| PL | - | - | - |
| Decision | - | - | SAT |

2.2.35 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg c, d\}$

Clause 2: $\{\neg \neg d\}$

Clause 3: $\{a, \ln c\}$
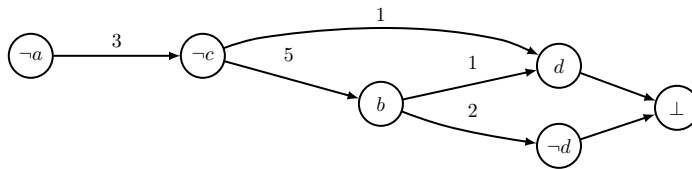
Clause 4: $\{\neg e\}$

Clause 5: $\{b, c\}$

Clause 6: $\{\neg a, \neg e\}$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg c$ | $\neg a, \neg c, b$ | $\neg a, \neg c, b, \neg d$ |
| Cl. 1: $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, d$ | $d$ | $\{\}$ ✗ |
| Cl. 2: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ |
| Cl. 3: $a, \neg c$ | $a, \neg c$ | $\neg c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $b, c$ | $b, c$ | $b, c$ | $b$ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg e$ | $\neg a, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | $\neg c$ | $b$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |

Conflict in step 5

$$\neg a \xrightarrow{3} \neg c \xrightarrow{5} b,\quad \neg c \xrightarrow{1} d,\quad b \xrightarrow{1} d,\quad b \xrightarrow{2} \neg d,\quad d \to \bot,\quad \neg d \to \bot$$

$$\dfrac{\dfrac{1.\ \neg b \vee c \vee d \quad 2.\ \neg b \vee \neg d}{\neg b \vee c} \quad 5.\ b \vee c}{\dfrac{c \quad 3.\ a \vee \neg c}{a}}$$

| Step | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg e$ | $a, \neg e, \neg c$ | $a, \neg e, \neg c, b$ | $a, \neg e, \neg c, b, \neg d$ |
| Cl. 1: $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, d$ | $d$ | $\{\}$ ✗ |
| Cl. 2: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ |
| Cl. 3: $a, \neg c$ | $a, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c$ | ✓ | ✓ | ✓ |
| Cl. 5: $b, c$ | $b, c$ | $b, c$ | $b, c$ | $b$ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg e$ | $\neg a, \neg e$ | $\neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $\neg e$ | $\neg c$ | $b$ | $\neg d$ | - |
| PL | - | - | - | - | - | - |
| Decision | - | - | - | - | - | UNSAT |

Conflict in step 11

$$\xrightarrow{7} a \xrightarrow{6} \neg e \xrightarrow{4} \neg c \xrightarrow{5} b,\quad \neg c \xrightarrow{1} d,\quad b \xrightarrow{1} d,\quad b \xrightarrow{2} \neg d,\quad d \to \bot,\quad \neg d \to \bot$$

$$\dfrac{\dfrac{\dfrac{\dfrac{1.\ \neg b \vee c \vee d \quad 2.\ \neg b \vee \neg d}{\neg b \vee c} \quad 5.\ b \vee c}{c \quad 4.\ \neg c \vee e}}{e \quad 6.\ \neg a \vee \neg e}}{\dfrac{\neg a \quad 7.\ a}{\bot}}$$

2.2.36 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{b, d\}$

Clause 2: $\{b, c\}$

Clause 3: $\{\neg b, \neg e\}$

Clause 4: $\{\neg a, \neg c\}$

Clause 5: $\{\neg c, \neg d\}$

Clause 6: $\{\neg b, c\}$

Clause 7: $\{a, b\}$

Clause 8: $\{\neg b, d, e\}$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c,$ $\neg d$ | $\neg a, b, c,$ $\neg d, \neg e$ |
| Cl. 1: $b, d$ | $b, d$ | $b, d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $b, c$ | $b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg e$ | $\neg e$ | $\neg e$ | ✓ |
| Cl. 4: $\neg a, \neg c$ | $\neg a, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg d$ | ✓ | ✓ |
| Cl. 6: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 7: $a, b$ | $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | $d, e$ | $d, e$ | $e$ | {} ✗ |
| BCP | - | $b$ | $c$ | $\neg d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - | - |

Conflict in step 6



$$\frac{\dfrac{8.\ \neg b \vee d \vee e \qquad 5.\ \neg c \vee \neg d}{\neg b \vee e \vee \neg c} \qquad 3.\ \neg b \vee \neg e}{\dfrac{\neg b \vee \neg c \qquad 6.\ \neg b \vee c}{\dfrac{\neg b \qquad 7.\ a \vee b}{a}}}$$

| Step | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg c$ | $a, \neg c, \neg b$ |
| Cl. 1: $b, d$ | $b, d$ | $b, d$ | $b, d$ | $d$ |
| Cl. 2: $b, c$ | $b, c$ | $b, c$ | $b$ | {} ✗ |
| Cl. 3: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | ✓ |
| Cl. 4: $\neg a, \neg c$ | $\neg a, \neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | ✓ | ✓ |
| Cl. 6: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $\neg b$ | ✓ |
| Cl. 7: $a, b$ | $a, b$ | ✓ | ✓ | ✓ |
| Cl. 8: $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ |
| BCP | $a$ | $\neg c$ | $\neg b$ | - |
| PL | - | - | - | - |
| Decision | - | - | - | UNSAT |

Conflict in step 10



$$\frac{\dfrac{2.\ b \vee c \qquad 6.\ \neg b \vee c}{c} \qquad 4.\ \neg a \vee \neg c}{\dfrac{\neg a \qquad 9.\ a}{\bot}}$$

2.2.37 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the

learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model.  If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg b, d, e\}$

Clause 2: $\{b, e\}$

Clause 3: $\{c, d\}$

Clause 4: $\{\neg a, \neg e\}$
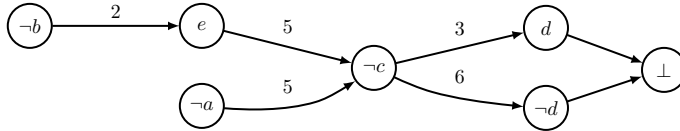
Clause 5: $\{a, \neg c, \neg e\}$

Clause 6: $\{c, \neg d\}$

Clause 7: $\{\neg b, \neg d\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, e$ | $\neg a, \neg b, e,$ $\neg c$ | $\neg a, \neg b, e,$ $\neg c, \neg d$ |
| Cl. 1: $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $b, e$ | $b, e$ | $b, e$ | $e$ | ✓ | ✓ | ✓ |
| Cl. 3: $c, d$ | $c, d$ | $c, d$ | $c, d$ | $c, d$ | $d$ | {} ✗ |
| Cl. 4: $\neg a, \neg e$ | $\neg a, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $a, \neg c, \neg e$ | $a, \neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c$ | ✓ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $\neg d$ | ✓ |
| Cl. 7: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | $e$ | $\neg c$ | $\neg d$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |

Conflict in step 6



$$\frac{\dfrac{3.\ c \vee d \qquad 6.\ c \vee \neg d}{c} \qquad 5.\ a \vee \neg c \vee \neg e}{\dfrac{a \vee \neg e \qquad\qquad 2.\ b \vee e}{a \vee b}}$$

| Step | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg d$ | $\neg a, b, \neg d,$ $c$ | $\neg a, b, \neg d,$ $c, \neg e$ |
| Cl. 1: $\neg b, d, e$ | $\neg b, d, e$ | $d, e$ | $e$ | $e$ | {} ✗ |
| Cl. 2: $b, e$ | $b, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $c, d$ | $c, d$ | $c, d$ | $c$ | ✓ | ✓ |
| Cl. 4: $\neg a, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $a, \neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg e$ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ | ✓ | ✓ |
| Cl. 8: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $\neg d$ | $c$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

Conflict in step 11



$$\frac{\dfrac{\dfrac{5.\ a \vee \neg c \vee \neg e \qquad 1.\ \neg b \vee d \vee e}{a \vee \neg c \vee \neg b \vee d} \qquad 3.\ c \vee d}{\dfrac{a \vee \neg b \vee d \qquad\qquad 7.\ \neg b \vee \neg d}{a \vee \neg b}} \qquad 8.\ a \vee b}{a}$$

| Step | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg e$ | $a, \neg e, b$ | $a, \neg e, b, \neg d$ |
| Cl. 1: $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d$ | $d$ | {} ✗ |
| Cl. 2: $b, e$ | $b, e$ | $b, e$ | $b$ | ✓ | ✓ |
| Cl. 3: $c, d$ | $c, d$ | $c, d$ | $c, d$ | $c, d$ | $c$ |
| Cl. 4: $\neg a, \neg e$ | $\neg a, \neg e$ | $\neg e$ | ✓ | ✓ | ✓ |
| Cl. 5: $a, \neg c, \neg e$ | $a, \neg c, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | ✓ |
| Cl. 7: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ |
| Cl. 8: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $\neg e$ | $b$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |

Conflict in step 16



2.2.38 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(a \vee b \vee c)$

Clause 2: $(\neg a \vee b)$

Clause 3: $(\neg b \vee c)$
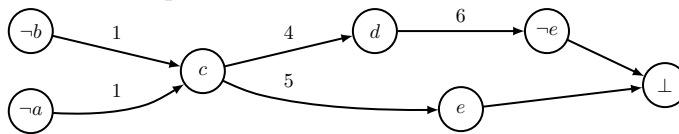
Clause 4: $(\neg c \vee d)$

Clause 5: $(\neg c \vee e)$

Clause 6: $(\neg d \vee \neg e)$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, d$ | $\neg a, \neg b, c, d, \neg e$ |
| Cl. 1: $a,b,c$ | $a,b,c$ | $b,c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a,b$ | $\neg a,b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b,c$ | $\neg b,c$ | $\neg b,c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c,d$ | $\neg c,d$ | $\neg c,d$ | $\neg c,d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c,e$ | $\neg c,e$ | $\neg c,e$ | $\neg c,e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| BCP | - | - | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |

Conflict in step 6



$$\frac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\dfrac{\neg c}{a \vee b} \qquad 1.\ a \vee b \vee c}$$

| Step | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, \neg e$ |
| Cl. 1: $a,b,c$ | $b,c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a,b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b,c$ | $\neg b,c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c,d$ | $\neg c,d$ | $\neg c,d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c,e$ | $\neg c,e$ | $\neg c,e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| Cl. 7: $a,b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

Conflict in step 11



$$\frac{\dfrac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\neg c} \qquad 3.\ \neg b \vee c}{\dfrac{\neg b}{a} \qquad 7.\ a \vee b}$$

| Step | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, b$ | $a, b, c$ | $a, b, c,$ $d$ | $a, b, c,$ $d, \neg e$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | $\neg a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | $\{\}$ ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| Cl. 7: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $b$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | - | - | - | - | - | UNSAT |

Conflict in step 17



2.2.39 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, \neg b, c\}$

Clause 2: $\{b, \neg c, d\}$

Clause 3: $\{a, \neg b\}$

Clause 4: $\{a, c\}$

Clause 5: $\{\neg c, \neg d\}$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, b$ | $a, b, \neg c$ |
| Cl. 1: $a, \neg b, c$ | $a, \neg b, c$ | ✓ | ✓ | ✓ |
| Cl. 2: $b, \neg c, d$ | $b, \neg c, d$ | $b, \neg c, d$ | ✓ | ✓ |
| Cl. 3: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ |
| Cl. 4: $a, c$ | $a, c$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | ✓ |
| BCP | - | - | - | - |
| PL | $a$ | $b$ | $\neg c$ | - |
| Decision | - | - | - | SAT |

2.2.40 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b, c\}$

Clause 2: $\{\neg b, \neg c, e\}$

Clause 3: $\{b, e\}$

Clause 4: $\{b, \neg d\}$

Clause 5: $\{\neg c, d\}$

Clause 6: $\{\neg c, e\}$

Clause 7: $\{\neg a, \neg b, \neg c\}$

Clause 8: $\{a, c, \neg e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, \neg d$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | $b, c$ | $c$ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $a, c, \neg e$ | $a, c, \neg e$ | $c, \neg e$ | $c, \neg e$ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg c, e$ | $\neg b, \neg c, e$ | $\neg b, \neg c, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $b, e$ | $b, e$ | $b, e$ | $e$ | $e$ | $e$ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $\neg d$ | $\neg d$ | ✓ |
| Cl. 7: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | $\{\}$ ✗ |
| Cl. 8: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ |
| BCP | - | - | $c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - |



$$\frac{\dfrac{7.\ \neg c \vee d \qquad 6.\ b \vee \neg d}{b \vee \neg c} \qquad 1.\ a \vee b \vee c}{a \vee b}$$

| Step | (1) | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 2 | 2 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, d$ | $\neg a, b, d, \neg c$ | $\neg a, b, d, \neg c, \neg e$ |
| Cl. 1: $a, b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| Cl. 4: $\neg a, \neg b, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $a, c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $\neg e$ | ✓ |
| Cl. 6: $\neg b, \neg c, e$ | $\neg b, \neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| Cl. 7: $b, e$ | $b, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | - | - | $\neg e$ | - |
| PL | - | $d$ | - | - | - |
| Decision | - | - | $\neg c$ | - | SAT |

2.2.41 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, c\}$

Clause 2: $\{\neg a, b, \neg c\}$

Clause 3: $\{\neg b, e\}$

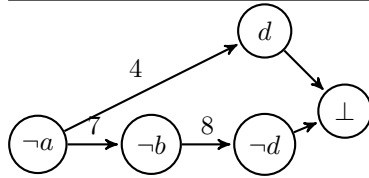Clause 4: $\{a, d\}$

Clause 5: $\{a, \neg c\}$

Clause 6: $\{\neg a, \neg e\}$

Clause 7: $\{a, \neg b\}$
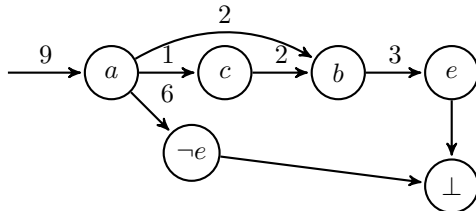
Clause 8: $\{b, \neg d\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c, \neg d$ |
| Cl. 1: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b, \neg c$ | $\neg a, b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | ✓ | ✓ | ✓ |
| Cl. 4: $a, d$ | $a, d$ | $d$ | $d$ | $d$ | {} ✗ |
| Cl. 5: $a, \neg c$ | $a, \neg c$ | $\neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg e$ | $\neg a, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a, \neg b$ | $a, \neg b$ | $\neg b$ | ✓ | ✓ | ✓ |
| Cl. 8: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $\neg d$ | $\neg d$ | ✓ |
| BCP | - | $\neg b$ | $\neg c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |



$$\frac{\dfrac{8.\ b \vee \neg d \qquad 4.\ a \vee d}{a \vee b} \qquad 7.\ a \vee \neg b}{a}$$

| Step | (1) | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, b$ | $a, c, b, \neg e$ |
| Cl. 1: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b, \neg c$ | $\neg a, b, \neg c$ | $b, \neg c$ | $b$ | ✓ | ✓ |
| Cl. 3: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $e$ | {} ✗ |
| Cl. 4: $a, d$ | $a, d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $a, \neg c$ | $a, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg e$ | $\neg a, \neg e$ | $\neg e$ | $\neg e$ | $\neg e$ | ✓ |
| Cl. 7: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | $b$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |



$$\frac{\dfrac{\dfrac{\dfrac{3.\ \neg b \vee e \qquad 6.\ \neg a \vee \neg e}{\neg a \vee \neg b} \qquad 2.\ \neg a \vee b \vee \neg c}{\neg a \vee \neg c} \qquad 1.\ \neg a \vee c}{\neg a} \qquad 9.\ a}{\bot}$$

2.2.42 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b, c\}$

Clause 2: $\{\neg a, b\}$

Clause 3: $\{\neg b, c\}$

Clause 4: $\{\neg c, d\}$

Clause 5: $\{\neg c, e\}$

Clause 6: $\{\neg d, \neg e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, d$ | $\neg a, \neg b, c, d, e$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | $b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| BCP | - | - | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |



$$\frac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\dfrac{\neg c}{a \vee b}} \qquad 1.\ a \vee b \vee c$$

| Step | (2) | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, \neg e$ |
| Cl. 1: $a, b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| Cl. 7: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

$$\frac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\dfrac{\dfrac{\neg c}{\neg b} \qquad 3.\ \neg b \vee c}{\dfrac{\neg b \qquad 7.\ a \vee b}{a}}}$$

| Step | (1) | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a,b$ | $a,b,c$ | $a,b,c,d$ | $a,b,c,d,\neg e$ |
| Cl. 1: $a,b,c$ | $a,b,c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a,b$ | $\neg a,b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b,c$ | $\neg b,c$ | $\neg b,c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c,d$ | $\neg c,d$ | $\neg c,d$ | $\neg c,d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c,e$ | $\neg c,e$ | $\neg c,e$ | $\neg c,e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg e$ | $\neg e$ | ✓ |
| Cl. 7: $a,b$ | $a,b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $b$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | - | - | - | - | - | UNSAT |



$$\frac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\dfrac{\dfrac{\neg c}{\neg b} \qquad 3.\ \neg b \vee c}{\dfrac{\dfrac{\neg b \qquad 2.\ \neg a \vee b}{\neg a} \qquad 8.\ a}{\bot}}}$$

2.2.43 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, \neg c, \neg e\}$

Clause 2: $\{\neg a, \neg e\}$

Clause 3: $\{b, e\}$

Clause 4: $\{\neg b, d, e\}$

Clause 5: $\{\neg b, \neg d\}$
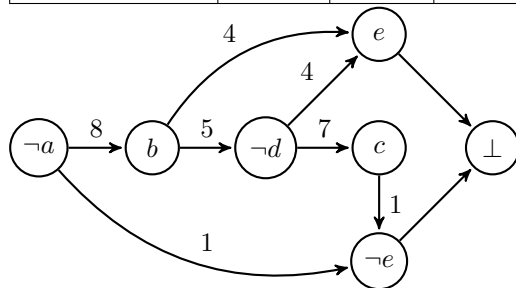
Clause 6: $\{c, \neg d\}$

Clause 7: $\{c, d\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, e$ | $\neg a, \neg b, e,$ $\neg c$ | $\neg a, \neg b, e,$ $\neg c, \neg d$ |
| Cl. 1: $a, \neg c, \neg e$ | $a, \neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c$ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg e$ | $\neg a, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, e$ | $b, e$ | $b, e$ | $e$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $\neg d$ | ✓ |
| Cl. 7: $c, d$ | $c, d$ | $c, d$ | $c, d$ | $c, d$ | $d$ | {} ✗ |
| BCP | - | - | $e$ | $\neg c$ | $\neg d$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |
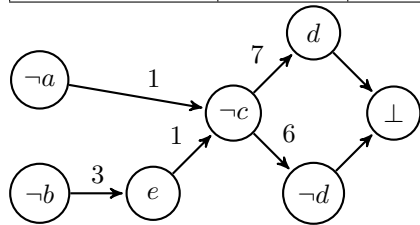
$$\frac{\dfrac{6.\ c \vee \neg d \qquad 7.\ c \vee d}{c} \qquad 1.\ a \vee \neg c \vee \neg e}{\dfrac{a \vee \neg e \qquad\qquad 3.\ b \vee e}{a \vee b}}$$

| Step | (2) | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg d$ | $\neg a, b, \neg d,$ $c$ | $\neg a, b, \neg d,$ $c, \neg e$ |
| Cl. 1: $a, \neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg e$ | ✓ |
| Cl. 2: $\neg a, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, e$ | $b, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, d, e$ | $\neg b, d, e$ | $d, e$ | $e$ | $e$ | {} ✗ |
| Cl. 5: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ | ✓ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $c, d$ | $c, d$ | $c, d$ | $c$ | ✓ | ✓ |
| Cl. 8: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $\neg d$ | $c$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{1.\ a \vee \neg c \vee \neg e \qquad 4.\ \neg b \vee d \vee \vee e}{a \vee \neg b \vee \neg c \vee d} \qquad 7.\ c \vee d}{a \vee \neg b \vee d} \qquad 5.\ \neg b \vee \neg d}{a \vee \neg b} \qquad 8.\ a \vee b}{a}}{}$$

| Step | (1) | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg e$ | $a, \neg e, b$ | $a, \neg e,$ $b, \neg d$ |
| Cl. 1: $a, \neg c, \neg e$ | $a, \neg c, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg e$ | $\neg a, \neg e$ | $\neg e$ | ✓ | ✓ | ✓ |
| Cl. 3: $b, e$ | $b, e$ | $b, e$ | $b$ | ✓ | ✓ |
| Cl. 4: $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d, e$ | $\neg b, d$ | $d$ | {} ✗ |
| Cl. 5: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | ✓ |
| Cl. 7: $c, d$ | $c, d$ | $c, d$ | $c, d$ | $c, d$ | $c$ |
| Cl. 8: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $\neg e$ | $b$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |



$$\frac{\dfrac{\dfrac{\dfrac{5.\ \neg b \vee \neg d \qquad 4.\ \neg b \vee d \vee \vee e}{\neg b \vee e} \qquad 3.\ b \vee e}{e} \qquad 2.\ \neg a \vee \neg e}{\neg a} \qquad 8.\ a}{\bot}$$

2.2.44 Use the DPLL algorithm with BCP to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase.
If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(\neg x \vee y)$

Clause 2: $(\neg y \vee \neg z)$

Clause 3: $(x \vee \neg w)$

Clause 4: $(w \vee z)$

Clause 5: $(\neg z \vee u)$

Clause 6: $(\neg u \vee \neg x)$

---

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg u$ | $\neg u, \neg z$ | $\neg u, \neg z, w$ | $\neg u, \neg z, w,$ $x$ | $\neg u, \neg z, w,$ $x, y$ |
| Cl. 1: $\neg x, y$ | $\neg x, y$ | $\neg x, y$ | $\neg x, y$ | $\neg x, y$ | $y$ | ✓ |
| Cl. 2: $\neg y, \neg z$ | $\neg y, \neg z$ | $\neg y, \neg z$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $x, \neg w$ | $x, \neg w$ | $x, \neg w$ | $x, \neg w$ | $x$ | ✓ | ✓ |
| Cl. 4: $w, z$ | $w, z$ | $w, z$ | $w$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg z, u$ | $\neg z, u$ | $\neg z$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg u, \neg x$ | $\neg u, \neg x$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | $\neg z$ | $w$ | $x$ | $y$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg u$ | - | - | - | - | SAT |

**2.2.45** Use the DPLL algorithm with BCP to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(\neg a \lor b)$

Clause 2: $(\neg b \lor c \lor d)$

Clause 3: $(a \lor \neg c)$

Clause 4: $(\neg d \lor e)$

Clause 5: $(\neg e \lor f)$

Clause 6: $(\neg f \lor g)$

Clause 7: $(\neg g)$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| Assignment | - | $\neg g$ | $\neg g, \neg f$ | $\neg g, \neg f, \neg e$ | $\neg g, \neg f, \neg e,$ $\neg d$ | $\neg g, \neg f, \neg e,$ $\neg d, \neg a$ | $\neg g, \neg f, \neg e,$ $\neg d, \neg a, \neg c$ | $\neg g, \neg f, \neg e,$ $\neg d, \neg a, \neg c,$ $\neg b$ |
| Cl. 1: $\neg a, b$ | $\neg a, b$ | $\neg a, b$ | $\neg a, b$ | $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c$ | $\neg b, c$ | $\neg b$ | ✓ |
| Cl. 3: $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 4: $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg f, g$ | $\neg f, g$ | $\neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg g$ | $\neg g$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $\neg g$ | $\neg f$ | $\neg e$ | $\neg d$ | - | $\neg c$ | $\neg b$ | - |
| PL | - | - | - | - | - | - | - | - |
| Decision | - | - | - | - | $\neg a$ | - | - | SAT |

**2.2.46** Use the DPLL algorithm with no explicit heuristics to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(a \lor b \lor c)$

---

Clause 2: (¬a ∨¬b)

Clause 3: (¬a ∨¬c)

Clause 4: (b ∨¬c)

Clause 5: (¬b ∨c)

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | $b, c$ | $c$ | {} ✗ |
| Cl. 2: $\neg a, \neg b$ | $\neg a, \neg b$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg c$ | $\neg a, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 4: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $\neg c$ | ✓ |
| Cl. 5: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ | ✓ |
| BCP | - | - | - | - |
| PL | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | $\neg c$ | - |

| Step | 5 | 6 |
|---|---|---|
| Decision Level | 2 | 3 |
| Assignment | $\neg a, \neg b$ | $\neg a, \neg b, c$ |
| Cl. 1: $a, b, c$ | $c$ | ✓ |
| Cl. 2: $\neg a, \neg b$ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg c$ | ✓ | ✓ |
| Cl. 4: $b, \neg c$ | $\neg c$ | {} ✗ |
| Cl. 5: $\neg b, c$ | ✓ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $c$ | - |

| Step | 7 | 8 | 9 |
|---|---|---|---|
| Decision Level | 1 | 2 | 3 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg c$ |
| Cl. 1: $a, b, c$ | $b, c$ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 4: $b, \neg c$ | $b, \neg c$ | ✓ | ✓ |
| Cl. 5: $\neg b, c$ | $\neg b, c$ | $c$ | {} ✗ |
| BCP | - | - | - |
| PL | - | - | - |
| Decision | $b$ | $\neg c$ | - |

| Step | 10 | 11 |
|---|---|---|
| Decision Level | 2 | 3 |
| Assignment | $\neg a, b$ | $\neg a, b, c$ |
| Cl. 1: $a, b, c$ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b$ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg c$ | ✓ | ✓ |
| Cl. 4: $b, \neg c$ | ✓ | ✓ |
| Cl. 5: $\neg b, c$ | $c$ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $c$ | SAT |

2.2.47 Use the DPLL algorithm with no explicit heuristics to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: (a ∨b)

Clause 2: (¬a ∨c)

Clause 3: (¬b ∨¬c)

Clause 4: (c ∨d)

Clause 5: (¬d ∨¬a)

Clause 6: (b ∨¬d)

Clause 7: (¬c ∨¬d)

**Solution**

| Step | 1 | 2 | 3 |
|---|---|---|---|
| Decision Level | 0 | 1 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ |
| Cl. 1: $a, b$ | $a, b$ | $b$ | $\{\}$ ✗ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ |
| Cl. 3: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | ✓ |
| Cl. 4: $c, d$ | $c, d$ | $c, d$ | $c, d$ |
| Cl. 5: $\neg d, \neg a$ | $\neg d, \neg a$ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $\neg d$ |
| Cl. 7: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ |
| BCP | - | - | - |
| PL | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - |

| Step | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| Decision Level | 1 | 2 | 3 | 4 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg c$ | $\neg a, b, \neg c, \neg d$ |
| Cl. 1: $a, b$ | $b$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 4: $c, d$ | $c, d$ | $c, d$ | $d$ | $\{\}$ ✗ |
| Cl. 5: $\neg d, \neg a$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | ✓ | ✓ |
| BCP | - | - | - | - |
| PL | - | - | - | - |
| Decision | $b$ | $\neg c$ | $\neg d$ | - |

| Step | 8 | 9 |
|---|---|---|
| Decision Level | 3 | 4 |
| Assignment | $\neg a, b, \neg c$ | $\neg a, b, \neg c, d$ |
| Cl. 1: $a, b$ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | ✓ | ✓ |
| Cl. 3: $\neg b, \neg c$ | ✓ | ✓ |
| Cl. 4: $c, d$ | $d$ | ✓ |
| Cl. 5: $\neg d, \neg a$ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | ✓ | ✓ |
| Cl. 7: $\neg c, \neg d$ | ✓ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $d$ | SAT |

2.2.48 Use the DPLL algorithm with BCP and PL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1:  (a ∨b)

Clause 2:  (¬a ∨c)

Clause 3:  (d ∨e)

Clause 4:  (¬b ∨¬c)

Clause 5:  (e ∨¬f)

Clause 6:  (¬d ∨¬f)

Clause 7:  (d ∨¬f)

Clause 8:  (b ∨¬e)

Clause 9:  (a ∨¬f)

### Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| Assignment | - | $\neg f$ | $\neg f, d$ | $\neg f, d, \neg e$ | $\neg f, d, \neg e,$ $\neg a$ | $\neg f, d, \neg e,$ $\neg a, b$ | $\neg f, d, \neg e,$ $\neg a, b, \neg c$ |
| Cl. 1: $a, b$ | $a, b$ | $a, b$ | $a, b$ | $a, b$ | $b$ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ |
| Cl. 3: $d, e$ | $d, e$ | $d, e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg c$ | ✓ |
| Cl. 5: $e, \neg f$ | $e, \neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg d, \neg f$ | $\neg d, \neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $d, \neg f$ | $d, \neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $b, \neg e$ | $b, \neg e$ | $b, \neg e$ | $b, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a, \neg f$ | $a, \neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | $b$ | $\neg c$ | - |
| PL | $\neg f$ | $d$ | $\neg e$ | - | - | - | - |
| Decision | - | - | - | $\neg a$ | - | - | SAT |

2.2.49  Use the DPLL algorithm with BCP, PL and CDCL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1:  (x ∨y ∨w)

Clause 2:  (¬x ∨z)

Clause 3:  (¬y ∨w)

Clause 4:  (¬z ∨¬w)

Clause 5:  (¬y)

Clause 6:  (¬x ∨w)

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 |
| Assignment | - | $\neg y$ | $\neg y, \neg w$ | $\neg y, \neg w, \neg x$ |
| Cl. 1: $x, y, w$ | $x, y, w$ | $x, w$ | $x$ | $\{\}$ ✗ |
| Cl. 2: $\neg x, z$ | $\neg x, z$ | $\neg x, z$ | $\neg x, z$ | ✓ |
| Cl. 3: $\neg y, w$ | $\neg y, w$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg z, \neg w$ | $\neg z, \neg w$ | $\neg z, \neg w$ | ✓ | ✓ |
| Cl. 5: $\neg y$ | $\neg y$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg x, w$ | $\neg x, w$ | $\neg x, w$ | $\neg x$ | ✓ |
| BCP | $\neg y$ | - | $\neg x$ | - |
| PL | - | - | - | - |
| Decision | - | $\neg w$ | - | - |

Conflict in step 4



$$\frac{\dfrac{1.\ x \vee y \vee w \qquad 5.\ \neg y}{x \vee w} \qquad 6.\ \neg x \vee w}{w}$$

| Step | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | $\neg y$ | $\neg y, w$ | $\neg y, w, \neg z$ | $\neg y, w, \neg z, \neg x$ |
| Cl. 1: $x, y, w$ | $x, w$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg x, z$ | $\neg x, z$ | $\neg x, z$ | $\neg x$ | ✓ |
| Cl. 3: $\neg y, w$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg z, \neg w$ | $\neg z, \neg w$ | $\neg z$ | ✓ | ✓ |
| Cl. 5: $\neg y$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg x, w$ | $\neg x, w$ | ✓ | ✓ | ✓ |
| Cl. 7: $w$ | $w$ | ✓ | ✓ | ✓ |
| BCP | $w$ | $\neg z$ | $\neg x$ | - |
| PL | - | - | - | - |
| Decision | - | - | - | SAT |

2.2.50 Use the DPLL algorithm with BCP, PL and CDCL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(a \vee b)$

Clause 2: $(\neg a \vee c)$

Clause 3: $(c \vee d)$

Clause 4: $(\neg b \vee \neg c)$

Clause 5: $(d \vee e)$

Clause 6: $(\neg e \vee f)$

Clause 7: $(\neg d \vee \neg f)$

Clause 8:  (b ∨¬f)

Clause 9:  (¬b ∨e)

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg c$ | $\neg a, b, \neg c, d$ | $\neg a, b, \neg c, d, e$ | $\neg a, b, \neg c, d, e, \neg f$ |
| Cl. 1: $a, b$ | $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $c, d$ | $c, d$ | $c, d$ | $c, d$ | $d$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d, e$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $f$ | {} ✗ |
| Cl. 7: $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg f$ | $\neg f$ | ✓ |
| Cl. 8: $b, \neg f$ | $b, \neg f$ | $b, \neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $e$ | $e$ | $e$ | ✓ | ✓ |
| BCP | - | $b$ | $\neg c$ | $d$ | $e$ | $\neg f$ | - |
| PL | - | - | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - | - | - |

Conflict in step 7

$\neg a$ →1→ $b$ →4→ $\neg c$ →3→ $d$ →7→ $\neg f$ → $\bot$

$b$ →9→ $e$ →6→ $f$ → $\bot$

$$\frac{6.\ \neg e \vee f \qquad 7.\ \neg d \vee \neg f}{\neg e \vee \neg d \qquad 3.\ c \vee d}$$
$$\frac{\qquad\qquad}{\neg e \vee c \qquad 4.\ \neg b \vee \neg c}$$
$$\frac{\qquad\qquad}{\neg e \vee \neg b \qquad 9.\ \neg b \vee e}$$
$$\frac{\qquad\qquad}{\neg b \qquad\qquad 1.\ a \vee b}$$
$$\frac{\qquad\qquad}{a}$$

| Step | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, \neg b$ | $a, c, \neg b, \neg f$ | $a, c, \neg b, \neg f, \neg e$ | $a, c, \neg b, \neg f, \neg e, d$ |
| Cl. 1: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $c, d$ | $c, d$ | $c, d$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d$ | ✓ |
| Cl. 6: $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e, f$ | $\neg e$ | ✓ | ✓ |
| Cl. 7: $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | ✓ | ✓ | ✓ |
| Cl. 8: $b, \neg f$ | $b, \neg f$ | $b, \neg f$ | $b, \neg f$ | $\neg f$ | ✓ | ✓ | ✓ |
| Cl. 9: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 10: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | $\neg b$ | $\neg f$ | $\neg e$ | $d$ | - |
| PL | - | - | - | - | - | - | - |
| Decision | - | - | - | - | - | - | SAT |

2.2.51 Use the DPLL algorithm with BCP, PL and CDCL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is

unsatisfiable.

Clause 1: $(u \lor v)$

Clause 2: $(\neg u \lor w)$

Clause 3: $(v \lor \neg x)$

Clause 4: $(w \lor y)$

Clause 5: $(\neg v \lor \neg y)$

Clause 6: $(\neg w \lor z)$
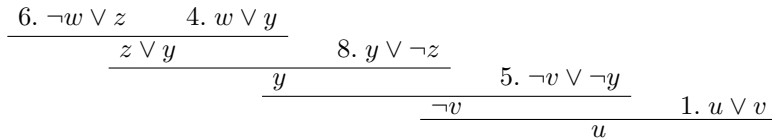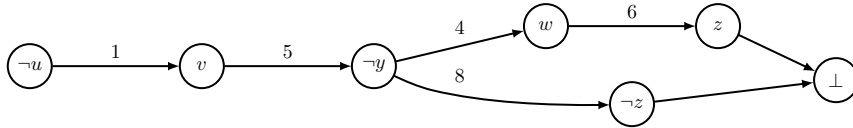
Clause 7: $(\neg z \lor x)$

Clause 8: $(y \lor \neg z)$
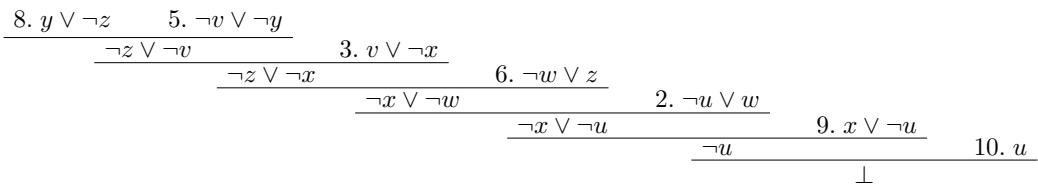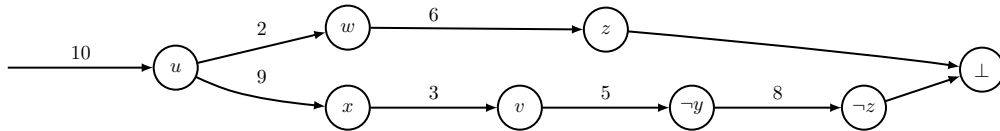
Clause 9: $(x \lor \neg u)$

## Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg u$ | $\neg u, v$ | $\neg u, v, \neg y$ | $\neg u, v, \neg y,$ $w$ | $\neg u, v, \neg y,$ $w, \neg z$ |
| Cl. 1: $u, v$ | $u, v$ | $v$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg u, w$ | $\neg u, w$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $w, y$ | $w, y$ | $w, y$ | $w, y$ | $w$ | ✓ | ✓ |
| Cl. 5: $\neg v, \neg y$ | $\neg v, \neg y$ | $\neg v, \neg y$ | $\neg y$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $z$ | {} ✗ |
| Cl. 7: $\neg z, x$ | $\neg z, x$ | $\neg z, x$ | $\neg z, x$ | $\neg z, x$ | $\neg z, x$ | ✓ |
| Cl. 8: $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $\neg z$ | $\neg z$ | ✓ |
| Cl. 9: $x, \neg u$ | $x, \neg u$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | $v$ | $\neg y$ | $w$ | $\neg z$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg u$ | - | - | - | - | - |

Conflict in step 6



$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{6.\ \neg w \vee z \quad 4.\ w \vee y}{z \vee y} \quad 8.\ y \vee \neg z}{y}}{\neg v} \quad 5.\ \neg v \vee \neg y}{} \quad 1.\ u \vee v}{u}$$

| Step | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $u$ | $u, w$ | $u, w, x$ | $u, w, x,$ $v$ | $u, w, x,$ $v, \neg y$ | $u, w, x,$ $v, \neg y, \neg z$ |
| Cl. 1: $u, v$ | $u, v$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg u, w$ | $\neg u, w$ | $w$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | $v$ | ✓ | ✓ | ✓ |
| Cl. 4: $w, y$ | $w, y$ | $w, y$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg v, \neg y$ | $\neg v, \neg y$ | $\neg v, \neg y$ | $\neg v, \neg y$ | $\neg v, \neg y$ | $\neg y$ | ✓ | ✓ |
| Cl. 6: $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $z$ | $z$ | $z$ | $z$ | {} ✗ |
| Cl. 7: $\neg z, x$ | $\neg z, x$ | $\neg z, x$ | $\neg z, x$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $\neg z$ | ✓ |
| Cl. 9: $x, \neg u$ | $x, \neg u$ | $x$ | $x$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 10: $u$ | $u$ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $u$ | $w$ | $x$ | $v$ | $\neg y$ | $\neg z$ | - |
| PL | - | - | - | - | - | - | - |
| Decision | - | - | - | - | - | - | UNSAT |

Conflict in step 13



$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{8.\ y \vee \neg z \quad 5.\ \neg v \vee \neg y}{\neg z \vee \neg v} \quad 3.\ v \vee \neg x}{\neg z \vee \neg x} \quad 6.\ \neg w \vee z}{\neg x \vee \neg w} \quad 2.\ \neg u \vee w}{\neg x \vee \neg u} \quad 9.\ x \vee \neg u}{\neg u} \quad 10.\ u}{\bot}$$

2.2.52 Use the DPLL algorithm (*without* BCP, PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in `SAT`, give a satisfying model.

Clause 1: $(a \vee b \vee c)$

Clause 2: $(a \vee \neg b \vee \neg c)$

Clause 3: $(\neg a \vee \neg b \vee c)$

Clause 4: $(a \vee b \vee \neg c)$

Clause 5: $(\neg c \vee \neg a)$

**Solution**

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 |
| Assignment | - | $a$ | $a, b$ | $a, b, c$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | ✓ | ✓ | ✓ |
| Cl. 2: $a, \neg b, \neg c$ | $a, \neg b, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg b, c$ | $\neg a, \neg b, c$ | $\neg b, c$ | $c$ | ✓ |
| Cl. 4: $a, b, \neg c$ | $a, b, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg a$ | $\neg c, \neg a$ | $\neg c$ | $\neg c$ | {} ✗ |
| BCP | - | - | - | - |
| PL | - | - | - | - |
| Decision | $a$ | $b$ | $c$ | - |

| Step | 5 | 6 |
|---|---|---|
| Decision Level | 2 | 3 |
| Assignment | $a, b$ | $a, b, \neg c$ |
| Cl. 1: $a, b, c$ | ✓ | ✓ |
| Cl. 2: $a, \neg b, \neg c$ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg b, c$ | $c$ | {} ✗ |
| Cl. 4: $a, b, \neg c$ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg a$ | $\neg c$ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $\neg c$ | - |

| Step | 7 | 8 | 9 |
|---|---|---|---|
| Decision Level | 1 | 2 | 3 |
| Assignment | $a$ | $a, \neg b$ | $a, \neg b, c$ |
| Cl. 1: $a, b, c$ | ✓ | ✓ | ✓ |
| Cl. 2: $a, \neg b, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg b, c$ | $\neg b, c$ | ✓ | ✓ |
| Cl. 4: $a, b, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg a$ | $\neg c$ | $\neg c$ | {} ✗ |
| BCP | - | - | - |
| PL | - | - | - |
| Decision | $\neg b$ | $c$ | - |

| Step | 10 | 11 |
|---|---|---|
| Decision Level | 2 | 3 |
| Assignment | $a, \neg b$ | $a, \neg b, \neg c$ |
| Cl. 1: $a, b, c$ | ✓ | ✓ |
| Cl. 2: $a, \neg b, \neg c$ | ✓ | ✓ |
| Cl. 3: $\neg a, \neg b, c$ | ✓ | ✓ |
| Cl. 4: $a, b, \neg c$ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg a$ | $\neg c$ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $\neg c$ | SAT |

**2.2.53** Use the DPLL algorithm (*without* BCP, PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. If the set of clauses resulted in SAT, give a satisfying model.

Clause 1: $(\neg a \vee \neg b)$

Clause 2: $(\neg a \vee \neg d)$

Clause 3: $(c \vee \neg b)$

Clause 4: $(\neg c \vee d)$

**Solution**

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ |
| Cl. 1: $\neg a, \neg b$ | $\neg a, \neg b$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg d$ | $\neg a, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 3: $c, \neg b$ | $c, \neg b$ | $c, \neg b$ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | ✓ |
| BCP | - | - | - | - |
| PL | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | $\neg c$ | SAT |

**2.2.54** Use the DPLL algorithm with *Boolean Constrain Propagation* (*without* PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in SAT, give a satisfying model.

Clause 1: $(\neg f \vee \neg b \vee \neg a)$

Clause 2: $(\neg e \vee a \vee \neg d)$

Clause 3: $(\neg a \vee c \vee b)$

Clause 4: $(f \vee \neg a \vee e)$

Clause 5: $(d \vee \neg a \vee \neg b)$

Clause 6: $(\neg a \vee \neg c \vee b)$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $a$ | $a, b$ | $a, b, d$ | $a, b, d, \neg f$ | $a, b, d, \neg f, e$ |
| Cl. 1: $\neg f, \neg b, \neg a$ | $\neg f, \neg b, \neg a$ | $\neg f, \neg b$ | $\neg f$ | $\neg f$ | ✓ | ✓ |
| Cl. 2: $\neg e, a, \neg d$ | $\neg e, a, \neg d$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, c, b$ | $\neg a, c, b$ | $c, b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $f, \neg a, e$ | $f, \neg a, e$ | $f, e$ | $f, e$ | $f, e$ | $e$ | ✓ |
| Cl. 5: $d, \neg a, \neg b$ | $d, \neg a, \neg b$ | $d, \neg b$ | $d$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg c, b$ | $\neg a, \neg c, b$ | $\neg c, b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | $d$ | $\neg f$ | $e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $a$ | $b$ | - | - | - | SAT |

**2.2.55** Use the DPLL algorithm with *Boolean Constrain Propagation* and *Pure Literals* (*without* clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in SAT, give a satisfying model.

Clause 1: $(\neg c \vee \neg d)$

Clause 2: $(a \vee \neg d \vee \neg e)$

Clause 3: $(e \vee \neg c)$

Clause 4: $(c \vee b)$

Clause 5: $(\neg b \vee \neg c)$

Clause 6: $(a \vee b)$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 1 | 1 |
| Assignment | - | $a$ | $a, \neg d$ | $a, \neg d, e$ | $a, \neg d, e,$ $b$ | $a, \neg d, e,$ $b, \neg c$ |
| Cl. 1: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, \neg d, \neg e$ | $a, \neg d, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $e, \neg c$ | $e, \neg c$ | $e, \neg c$ | $e, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 4: $c, b$ | $c, b$ | $c, b$ | $c, b$ | $c, b$ | ✓ | ✓ |
| Cl. 5: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg c$ | ✓ |
| Cl. 6: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | $\neg c$ | - |
| PL | $a$ | $\neg d$ | $e$ | - | - | - |
| Decision | - | - | - | $b$ | - | SAT |

**2.2.56** Use the DPLL algorithm with *Boolean Constrain Propagation* (*without* PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. If the set of clauses resulted in SAT, give a satisfying model.

Clause 1: $(\neg a \vee \neg b)$

Clause 2: $(\neg a \vee c)$

Clause 3: $(\neg b \vee \neg c)$

Clause 4: $(b \vee c)$

Clause 5: $(a \vee \neg b)$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 |
| Assignment | - | $a$ | $a, \neg b$ | $a, \neg b, c$ |
| Cl. 1: $\neg a, \neg b$ | $\neg a, \neg b$ | $\neg b$ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | $c$ | $c$ | ✓ |
| Cl. 3: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | ✓ | ✓ |
| Cl. 4: $b, c$ | $b, c$ | $b, c$ | $c$ | ✓ |
| Cl. 5: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ |
| BCP | - | $\neg b$ | $c$ | - |
| PL | - | - | - | - |
| Decision | $a$ | - | - | SAT |

**2.2.57** Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in

UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, b, c\}$

Clause 2: $\{\neg b, \neg c, d\}$

Clause 3: $\{c, d, \neg e\}$

Clause 4: $\{\neg a, d, \neg e\}$

Clause 5: $\{a, b, d\}$

Clause 6: $\{c, \neg d, e\}$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, d$ | $\neg a, \neg b, d, c$ |
| Cl. 1: $\neg a, b, c$ | $\neg a, b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, \neg c, d$ | $\neg b, \neg c, d$ | $\neg b, \neg c, d$ | ✓ | ✓ | ✓ |
| Cl. 3: $c, d, \neg e$ | $c, d, \neg e$ | $c, d, \neg e$ | $c, d, \neg e$ | ✓ | ✓ |
| Cl. 4: $\neg a, d, \neg e$ | $\neg a, d, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $a, b, d$ | $a, b, d$ | $b, d$ | $d$ | ✓ | ✓ |
| Cl. 6: $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $c, e$ | ✓ |
| BCP | - | - | $d$ | - | - |
| PL | - | - | - | $c$ | - |
| Decision | $\neg a$ | $\neg b$ | - | - | SAT |

2.2.58 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, \neg b\}$

Clause 2: $\{\neg a, \neg c, \neg e\}$

Clause 3: $\{\neg c, d\}$

Clause 4: $\{\neg b, d\}$

Clause 5: $\{\neg d, e\}$

Clause 6: $\{\neg a, b\}$

Clause 7: $\{a, d, \neg e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 | 2 | 2 |
| Assignment | - | $\neg c$ | $\neg c, \neg a$ | $\neg c, \neg a, \neg b$ | $\neg c, \neg a, \neg b,$ $\neg d$ | $\neg c, \neg a, \neg b,$ $\neg d, \neg e$ |
| Cl. 1: $\neg a, \neg b$ | $\neg a, \neg b$ | $\neg a, \neg b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg c, \neg e$ | $\neg a, \neg c, \neg e$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg c, d$ | $\neg c, d$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | ✓ | ✓ |
| Cl. 6: $\neg a, b$ | $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a, d, \neg e$ | $a, d, \neg e$ | $a, d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $\neg e$ | ✓ |
| BCP | - | - | - | - | $\neg e$ | - |
| PL | $\neg c$ | - | $\neg b$ | - | - | - |
| Decision | - | $\neg a$ | - | $\neg d$ | - | SAT |

2.2.59 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, \neg c\}$

Clause 2: $\{b, c, e\}$

Clause 3: $\{b, e\}$

Clause 4: $\{\neg a, c\}$

Clause 5: $\{d, \neg e\}$

Clause 6: $\{b, \neg d\}$

Clause 7: $\{\neg d, \neg e\}$

Clause 8: $\{a, c\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 1 | 1 |
| Assignment | - | $b$ | $b, \neg e$ | $b, \neg e, \neg a$ | $b, \neg e, \neg a, \neg c$ |
| Cl. 1: $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $a, \neg c$ | $\neg c$ | ✓ |
| Cl. 2: $b, c, e$ | $b, c, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, e$ | $b, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | ✓ | ✓ |
| Cl. 5: $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | ✓ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | ✓ | ✓ | ✓ |
| Cl. 8: $a, c$ | $a, c$ | $a, c$ | $a, c$ | $c$ | $\{\}$ ✗ |
| BCP | - | - | - | $\neg c$ | - |
| PL | $b$ | $\neg e$ | - | - | - |
| Decision | - | - | $\neg a$ | - | - |

Conflict in step 5



$$\frac{1.\; a \vee \neg c \qquad 8.\; a \vee c}{a}$$

| Step | 6 | 7 | 8 |
|---|---|---|---|
| Decision Level | 0 | 0 | 0 |
| Assignment | $b, \neg e$ | $b, \neg e, a$ | $b, \neg e, a, c$ |
| Cl. 1: $a, \neg c$ | $a, \neg c$ | ✓ | ✓ |
| Cl. 2: $b, c, e$ | ✓ | ✓ | ✓ |
| Cl. 3: $b, e$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ |
| Cl. 5: $d, \neg e$ | ✓ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg d, \neg e$ | ✓ | ✓ | ✓ |
| Cl. 8: $a, c$ | $a, c$ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ |
| BCP | $a$ | $c$ | - |
| PL | - | - | - |
| Decision | - | - | SAT |

2.2.60 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{b, d\}$

Clause 2: $\{\neg a, c\}$

Clause 3: $\{a, \neg d\}$

Clause 4: $\{\neg b, c\}$

Clause 5: $\{\neg c, d\}$

Clause 6: $\{\neg c, e\}$

Clause 7: $\{\neg d, \neg e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg d$ | $\neg a, \neg d, b$ | $\neg a, \neg d, b,$ $\neg c$ |
| Cl. 1: $b, d$ | $b, d$ | $b, d$ | $b$ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $a, \neg d$ | $a, \neg d$ | $\neg d$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $c$ | {} ✗ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c$ | $\neg c$ | ✓ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | ✓ | ✓ | ✓ |
| BCP | - | $\neg d$ | $b$ | $\neg c$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |

Conflict in step 5



$$\frac{\dfrac{\dfrac{\dfrac{4.\ \neg b \vee c \qquad 1.\ b \vee d}{c \vee d} \qquad 5.\ \neg c \vee d}{d} \qquad 3.\ a \vee \neg d}{a}}{}$$

| Step | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, d$ | $a, c, d,$ $\neg e$ |
| Cl. 1: $b, d$ | $b, d$ | $b, d$ | $b, d$ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $a, \neg d$ | $a, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| Cl. 8: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |

Conflict in step 10



$$\frac{\dfrac{\dfrac{\dfrac{7.\ \neg d \vee \neg e \qquad 5.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 6.\ \neg c \vee e}{\neg c} \qquad 2.\ \neg a \vee c}{\neg a} \qquad 8.\ a}{\bot}$$

2.2.61 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the

learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg b, \neg e\}$

Clause 2: $\{a, d, e\}$

Clause 3: $\{b, \neg c\}$

Clause 4: $\{c, \neg d, e\}$
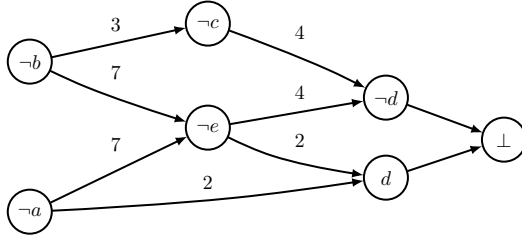
Clause 5: $\{\neg c, e\}$

Clause 6: $\{\neg a, c\}$

Clause 7: $\{a, b, \neg e\}$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c,$ $\neg e$ | $\neg a, \neg b, \neg c,$ $\neg e, \neg d$ |
| Cl. 1: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, d, e$ | $a, d, e$ | $d, e$ | $d, e$ | $d, e$ | $d$ | {} ✗ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $\neg c$ | ✓ | ✓ | ✓ |
| Cl. 4: $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $\neg d, e$ | $\neg d$ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a, b, \neg e$ | $a, b, \neg e$ | $b, \neg e$ | $\neg e$ | $\neg e$ | ✓ | ✓ |
| BCP | - | - | $\neg c$ | $\neg e$ | $\neg d$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |

Conflict in step 6



$$\cfrac{\cfrac{2.\ a \vee d \vee e \qquad 4.\ c \vee \neg d \vee e}{a \vee e \vee c} \qquad 7.\ a \vee b \vee \neg e}{\cfrac{a \vee c \vee b \qquad \qquad 3.\ b \vee \neg c}{a \vee b}}$$

| Step | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg e$ | $\neg a, b, \neg e,$ $\neg c$ | $\neg a, b, \neg e,$ $\neg c, \neg d$ |
| Cl. 1: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg e$ | ✓ | ✓ | ✓ |
| Cl. 2: $a, d, e$ | $d, e$ | $d, e$ | $d$ | $d$ | {} ✗ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d$ | $\neg d$ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c$ | ✓ | ✓ |
| Cl. 6: $\neg a, c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $a, b, \neg e$ | $b, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $\neg e$ | $\neg c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

Conflict in step 11



$$\cfrac{\cfrac{\cfrac{4.\ c \vee \neg d \vee e \qquad 2.\ a \vee d \vee e}{c \vee e \vee a} \qquad 5.\ \neg c \vee e}{e \vee a} \qquad 1.\ \neg b \vee \neg e}{\cfrac{a \vee \neg b \qquad \qquad 8.\ a \vee b}{a}}$$

| Step | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a,c$ | $a,c,b$ | $a,c,b,$ $\neg e$ |
| Cl. 1: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg e$ | ✓ |
| Cl. 2: $a,d,e$ | $a,d,e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $b$ | ✓ | ✓ |
| Cl. 4: $c, \neg d, e$ | $c, \neg d, e$ | $c, \neg d, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg a, c$ | $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ |
| Cl. 7: $a, b, \neg e$ | $a, b, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | $b$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |

Conflict in step 16



$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{1.\ \neg b \vee \neg e \qquad 3.\ b \vee \neg c}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\neg c} \qquad 6.\ \neg a \vee c}{\neg a} \qquad 9.\ a}{\bot}}{}$$

2.2.62 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(a \vee \neg b \vee c)$

Clause 2: $(\neg a \vee b)$

Clause 3: $(b \vee c)$
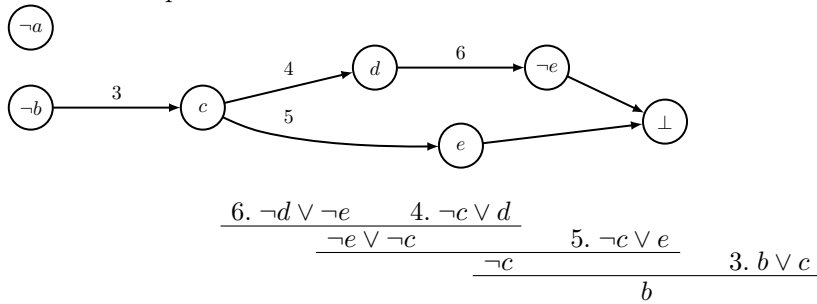
Clause 4: $(\neg c \vee d)$

Clause 5: $(\neg c \vee e)$
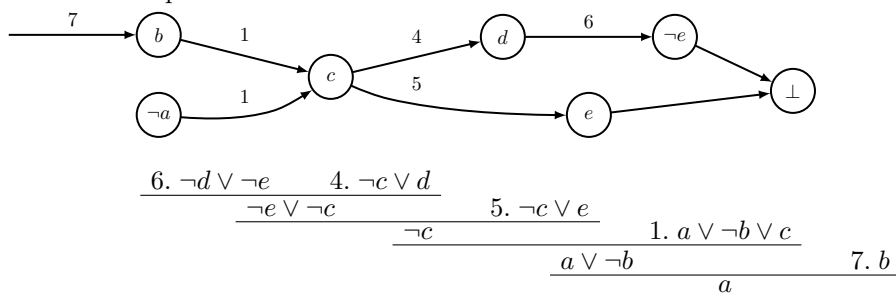
Clause 6: $(\neg d \vee \neg e)$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, d$ | $\neg a, \neg b, c, d, \neg e$ |
| Cl. 1: $a, \neg b, c$ | $a, \neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, c$ | $b, c$ | $b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| BCP | - | - | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |

Conflict in step 6



$$\frac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\dfrac{\neg c}{b} \qquad 3.\ b \vee c}$$

| Step | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, \neg e$ |
| Cl. 1: $a, \neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| Cl. 7: $b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

Conflict in step 11



$$\frac{\dfrac{\dfrac{6.\ \neg d \vee \neg e \qquad 4.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 5.\ \neg c \vee e}{\dfrac{\neg c}{a \vee \neg b} \qquad 1.\ a \vee \neg b \vee c}}{a} \qquad 7.\ b$$

| Step | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, b$ | $a, b, \neg c$ | $a, b, \neg c, \neg d$ |
| Cl. 1: $a, \neg b, c$ | $a, \neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | $\neg a, b$ | $b$ | ✓ | ✓ | ✓ |
| Cl. 3: $b, c$ | $b, c$ | $b, c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | ✓ |
| Cl. 7: $b$ | $b$ | $b$ | ✓ | ✓ | ✓ |
| Cl. 8: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $b$ | - | - | - |
| PL | - | - | $\neg c$ | $\neg d$ | - |
| Decision | - | - | - | - | SAT |

2.2.63 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg c, \neg d\}$

Clause 2: $\{c, e\}$

Clause 3: $\{\neg d, c\}$

Clause 4: $\{d, \neg e\}$

Clause 5: $\{b, \neg d\}$

Clause 6: $\{\neg d, \neg e\}$

Clause 7: $\{a, c\}$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 1 | 1 | 1 |
| Assignment | - | $a$ | $a,b$ | $a,b,\neg c$ | $a,b,\neg c,$ $\neg d$ | $a,b,\neg c,$ $\neg d,\neg e$ |
| Cl. 1: $\neg c,\neg d$ | $\neg c,\neg d$ | $\neg c,\neg d$ | $\neg c,\neg d$ | ✓ | ✓ | ✓ |
| Cl. 2: $c,e$ | $c,e$ | $c,e$ | $c,e$ | $e$ | $e$ | {} ✗ |
| Cl. 3: $\neg d,c$ | $\neg d,c$ | $\neg d,c$ | $\neg d,c$ | $\neg d$ | ✓ | ✓ |
| Cl. 4: $d,\neg e$ | $d,\neg e$ | $d,\neg e$ | $d,\neg e$ | $d,\neg e$ | $\neg e$ | ✓ |
| Cl. 5: $b,\neg d$ | $b,\neg d$ | $b,\neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg d,\neg e$ | ✓ | ✓ |
| Cl. 7: $a,c$ | $a,c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | $\neg d$ | $\neg e$ | - |
| PL | $a$ | $b$ | - | - | - | - |
| Decision | - | - | $\neg c$ | - | - | - |

Conflict in step 6

$\neg c \xrightarrow{2} e$
$\neg c \xrightarrow{3} \neg d \xrightarrow{4} \neg e$
$e \to \bot \quad \neg e \to \bot$

$$\frac{\dfrac{4.\ d \vee \neg e \qquad 2.\ c \vee e}{d \vee c} \qquad 3.\ \neg d \vee c}{c}$$

| Step | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | $a,b$ | $a,b,c$ | $a,b,c,$ $\neg d$ | $a,b,c,$ $\neg d,\neg e$ |
| Cl. 1: $\neg c,\neg d$ | $\neg c,\neg d$ | $\neg d$ | ✓ | ✓ |
| Cl. 2: $c,e$ | $c,e$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg d,c$ | $\neg d,c$ | ✓ | ✓ | ✓ |
| Cl. 4: $d,\neg e$ | $d,\neg e$ | $d,\neg e$ | $\neg e$ | ✓ |
| Cl. 5: $b,\neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg d,\neg e$ | $\neg d,\neg e$ | $\neg d,\neg e$ | ✓ | ✓ |
| Cl. 7: $a,c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $c$ | $c$ | ✓ | ✓ | ✓ |
| BCP | $c$ | $\neg d$ | $\neg e$ | - |
| PL | - | - | - | - |
| Decision | - | - | - | SAT |

2.2.64 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(a \vee b \vee \neg c)$

Clause 2: $(\neg a \vee b)$

Clause 3: $(b \vee c)$

Clause 4: $(\neg c \vee d)$

Clause 5: $(\neg c \vee e)$

Clause 6: $(\neg d \vee \neg e)$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | - | $b$ | $b, \neg c$ | $b, \neg c, \neg d$ |
| Cl. 1: $a, b, \neg c$ | $a, b, \neg c$ | ✔ | ✔ | ✔ |
| Cl. 2: $\neg a, b$ | $\neg a, b$ | ✔ | ✔ | ✔ |
| Cl. 3: $b, c$ | $b, c$ | ✔ | ✔ | ✔ |
| Cl. 4: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | ✔ | ✔ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✔ | ✔ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | ✔ |
| BCP | - | - | - | - |
| PL | $b$ | $\neg c$ | $\neg d$ | - |
| Decision | - | - | - | SAT |

**2.2.65** Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b, c\}$

Clause 2: $\{\neg b, \neg c, d\}$

Clause 3: $\{\neg c, d, \neg e\}$

Clause 4: $\{\neg a, d, \neg e\}$

Clause 5: $\{c, b, d\}$

Clause 6: $\{\neg a, \neg d, e\}$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, d$ | $\neg a, d, b$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | $b, c$ | $b, c$ | ✔ |
| Cl. 2: $\neg b, \neg c, d$ | $\neg b, \neg c, d$ | $\neg b, \neg c, d$ | ✔ | ✔ |
| Cl. 3: $c, d, \neg e$ | $c, d, \neg e$ | $c, d, \neg e$ | ✔ | ✔ |
| Cl. 4: $\neg a, d, \neg e$ | $\neg a, d, \neg e$ | ✔ | ✔ | ✔ |
| Cl. 5: $c, b, d$ | $c, b, d$ | $c, b, d$ | ✔ | ✔ |
| Cl. 6: $\neg a, \neg d, e$ | $\neg a, \neg d, e$ | ✔ | ✔ | ✔ |
| BCP | - | - | - | - |
| PL | - | $d$ | $b$ | - |
| Decision | $\neg a$ | - | - | SAT |

**2.2.66** Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.
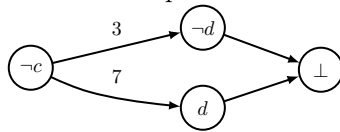
Clause 1: $\{\neg c, \neg d\}$

Clause 2: $\{c, e\}$

Clause 3: $\{\neg d, c\}$

Clause 4: $\{d, \neg e\}$

Clause 5: $\{\neg a, \neg e, \neg c\}$

Clause 6: $\{\neg d, \neg c\}$

Clause 7: $\{a, c\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg c$ | $\neg a, b, \neg c, \neg d$ |
| Cl. 1: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | ✓ | ✓ |
| Cl. 2: $c, e$ | $c, e$ | $c, e$ | $c, e$ | $e$ | $e$ |
| Cl. 3: $\neg d, c$ | $\neg d, c$ | $\neg d, c$ | $\neg d, c$ | $\neg d$ | ✓ |
| Cl. 4: $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $\neg e$ |
| Cl. 5: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg e, \neg c$ | $\neg a, \neg e, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $d, c$ | $d, c$ | $d, c$ | $d, c$ | $d$ | $\{\}$ ✗ |
| BCP | - | - | - | $\neg d$ | - |
| PL | $\neg a$ | $b$ | - | - | - |
| Decision | - | - | $\neg c$ | - | - |

Conflict in step 5



$$\frac{3.\ \neg d \vee c \qquad 7.\ d \vee c}{c}$$

| Step | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, \neg d$ | $\neg a, b, c, \neg d, \neg e$ |
| Cl. 1: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg d$ | ✓ | ✓ |
| Cl. 2: $c, e$ | $c, e$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg d, c$ | $\neg d, c$ | ✓ | ✓ | ✓ |
| Cl. 4: $d, \neg e$ | $d, \neg e$ | $d, \neg e$ | $\neg e$ | ✓ |
| Cl. 5: $b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg a, \neg e, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $d, c$ | $d, c$ | ✓ | ✓ | ✓ |
| Cl. 8: $c$ | $c$ | ✓ | ✓ | ✓ |
| BCP | $c$ | $\neg d$ | $\neg e$ | - |
| PL | - | - | - | - |
| Decision | - | - | - | SAT |

2.2.67 Use the DPLL algorithm with BCP, PL and CDCL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(x \vee y \vee w)$

Clause 2: $(\neg x \lor z \lor \neg w)$

Clause 3: $(\neg y \lor w)$

Clause 4: $(\neg z \lor \neg w)$

Clause 5: $(\neg y \lor z)$

Clause 6: $(\neg x \lor w)$

<br>

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 |
| Assignment | - | $\neg w$ | $\neg w, \neg x$ | $\neg w, \neg x, \neg y$ |
| Cl. 1: $x, y, w$ | $x, y, w$ | $x, y$ | $y$ | $\{\}$ ✗ |
| Cl. 2: $\neg x, z, \neg w$ | $\neg x, z, \neg w$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg y, w$ | $\neg y, w$ | $\neg y$ | $\neg y$ | ✓ |
| Cl. 4: $\neg z, \neg w$ | $\neg z, \neg w$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg y, z$ | $\neg y, z$ | $\neg y, z$ | $\neg y, z$ | ✓ |
| Cl. 6: $\neg x, w$ | $\neg x, w$ | $\neg x$ | ✓ | ✓ |
| BCP | - | $\neg x$ | $\neg y$ | - |
| PL | - | - | - | - |
| Decision | $\neg w$ | - | - | - |

Conflict in step 4

$$\dfrac{\dfrac{1.\ x \lor y \lor w \qquad 3.\ \neg y \lor w}{x \lor w} \qquad 6.\ \neg x \lor w}{w}$$

| Step | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $w$ | $w, \neg z$ | $w, \neg z, \neg x$ | $w, \neg z, \neg x, \neg y$ |
| Cl. 1: $x, y, w$ | $x, y, w$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg x, z, \neg w$ | $\neg x, z, \neg w$ | $\neg x, z$ | $\neg x$ | ✓ | ✓ |
| Cl. 3: $\neg y, w$ | $\neg y, w$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg z, \neg w$ | $\neg z, \neg w$ | $\neg z$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg y, z$ | $\neg y, z$ | $\neg y, z$ | $\neg y$ | $\neg y$ | ✓ |
| Cl. 6: $\neg x, w$ | $\neg x, w$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $w$ | $w$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $w$ | $\neg z$ | $\neg x$ | $\neg y$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | SAT |

**2.2.68** Use the DPLL algorithm with BCP, PL and CDCL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.
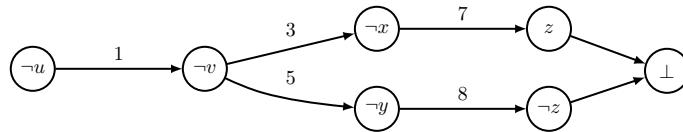
Clause 1: $(u \lor \neg v)$

Clause 2:  $(\neg u \lor w)$

Clause 3:  $(v \lor \neg x)$

Clause 4:  $(w \lor \neg y)$

Clause 5:  $(v \lor \neg y)$

Clause 6:  $(\neg w \lor z)$

Clause 7:  $(z \lor x)$

Clause 8:  $(y \lor \neg z)$

<div style="border:1px solid">

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg u$ | $\neg u, \neg v$ | $\neg u, \neg v, \neg x$ | $\neg u, \neg v, \neg x,$ $\neg y$ | $\neg u, \neg v, \neg x,$ $\neg y, \neg z$ |
| Cl. 1: $u, \neg v$ | $u, \neg v$ | $\neg v$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg u, w$ | $\neg u, w$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | $\neg x$ | ✓ | ✓ | ✓ |
| Cl. 4: $w, \neg y$ | $w, \neg y$ | $w, \neg y$ | $w, \neg y$ | $w, \neg y$ | ✓ | ✓ |
| Cl. 5: $v, \neg y$ | $v, \neg y$ | $v, \neg y$ | $\neg y$ | $\neg y$ | ✓ | ✓ |
| Cl. 6: $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $\neg w$ |
| Cl. 7: $z, x$ | $z, x$ | $z, x$ | $z, x$ | $z$ | $z$ | $\{\}$ ✗ |
| Cl. 8: $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $\neg z$ | ✓ |
| BCP | - | $\neg v$ | $\neg x$ | $\neg y$ | $\neg z$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg u$ | - | - | - | - | - |

Conflict in step 6



$$\cfrac{\cfrac{\cfrac{\cfrac{8.\ y \lor \neg z \qquad 7.\ z \lor x}{y \lor x} \qquad 3.\ v \lor \neg x}{y \lor v} \qquad 5.\ v \lor \neg y}{v} \qquad 1.\ u \lor \neg v}{u}$$

| Step | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $u$ | $u, w$ | $u, w, z$ | $u, w, z,$ $y$ | $u, w, z,$ $y, v$ |
| Cl. 1: $u, \neg v$ | $u, \neg v$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg u, w$ | $\neg u, w$ | $w$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | $v, \neg x$ | ✓ |
| Cl. 4: $w, \neg y$ | $w, \neg y$ | $w, \neg y$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $v, \neg y$ | $v, \neg y$ | $v, \neg y$ | $v, \neg y$ | $v, \neg y$ | $v$ | ✓ |
| Cl. 6: $\neg w, z$ | $\neg w, z$ | $\neg w, z$ | $z$ | ✓ | ✓ | ✓ |
| Cl. 7: $z, x$ | $z, x$ | $z, x$ | $z, x$ | ✓ | ✓ | ✓ |
| Cl. 8: $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y, \neg z$ | $y$ | ✓ | ✓ |
| Cl. 9: $u$ | $u$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | $u$ | $w$ | $z$ | $y$ | $v$ | - |
| PL | - | - | - | - | - | - |
| Decision | - | - | - | - | - | SAT |

</div>

2.2.69  Use the DPLL algorithm with BCP and PL to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: (a ∨b)

Clause 2: (a ∨c)

Clause 3: (d ∨e)

Clause 4: (¬b ∨¬c)

Clause 5: (e ∨¬f)

Clause 6: (¬d ∨¬f)

Clause 7: (d ∨¬f)

Clause 8: (b ∨e)

Clause 9: (a ∨¬f)

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg c$ | $a, \neg c, b$ | $a, \neg c, b,$ $e$ | $a, \neg c, b,$ $e, \neg f$ |
| Cl. 1: $a, b$ | $a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, c$ | $a, c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d, e$ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $e, \neg f$ | $e, \neg f$ | $e, \neg f$ | $e, \neg f$ | $e, \neg f$ | ✓ | ✓ |
| Cl. 6: $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | $\neg d, \neg f$ | ✓ |
| Cl. 7: $d, \neg f$ | $d, \neg f$ | $d, \neg f$ | $d, \neg f$ | $d, \neg f$ | $d, \neg f$ | ✓ |
| Cl. 8: $b, e$ | $b, e$ | $b, e$ | $b, e$ | ✓ | ✓ | ✓ |
| Cl. 9: $a, \neg f$ | $a, \neg f$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | - | - |
| PL | $a$ | $\neg c$ | $b$ | $e$ | $\neg f$ | - |
| Decision | - | - | - | - | - | SAT |

2.2.70 Use the DPLL algorithm (*without* BCP, PL and clause learning) to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. If the set of clauses resulted in `SAT`, give a satisfying model.

Clause 1: $(\neg a \vee \neg b)$

Clause 2: $(c \vee \neg d)$

Clause 3: $(a \vee \neg b)$

Clause 4: $(c \vee d)$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 3 | 4 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c, \neg d$ |
| Cl. 1: $\neg a, \neg b$ | $\neg a, \neg b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | $\neg d$ | ✓ |
| Cl. 3: $a, \neg b$ | $a, \neg b$ | $\neg b$ | ✓ | ✓ | ✓ |
| Cl. 4: $c, d$ | $c, d$ | $c, d$ | $c, d$ | $d$ | {} ✗ |
| BCP | - | - | - | - | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | $\neg c$ | $\neg d$ | - |

| Step | 6 | 7 |
|---|---|---|
| Decision Level | 3 | 4 |
| Assignment | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c, d$ |
| Cl. 1: $\neg a, \neg b$ | ✓ | ✓ |
| Cl. 2: $c, \neg d$ | $\neg d$ | {} ✗ |
| Cl. 3: $a, \neg b$ | ✓ | ✓ |
| Cl. 4: $c, d$ | $d$ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $d$ | - |

| Step | 8 | 9 |
|---|---|---|
| Decision Level | 2 | 3 |
| Assignment | $\neg a, \neg b$ | $\neg a, \neg b, c$ |
| Cl. 1: $\neg a, \neg b$ | ✓ | ✓ |
| Cl. 2: $c, \neg d$ | $c, \neg d$ | ✓ |
| Cl. 3: $a, \neg b$ | ✓ | ✓ |
| Cl. 4: $c, d$ | $c, d$ | ✓ |
| BCP | - | - |
| PL | - | - |
| Decision | $c$ | SAT |

2.2.71 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $(a \vee \neg b \vee c)$

Clause 2: $(\neg a \vee \neg b \vee \neg c)$

Clause 3: $(a \vee c \vee \neg e)$

Clause 4: $(b \vee \neg c \vee e)$

Clause 5: $(c \vee e)$

Clause 6: $(b \vee \neg d)$

Clause 7: $(\neg b \vee d)$

Clause 8: $(\neg c \vee e)$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 3 | 3 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg d$ | $\neg a, \neg b, \neg d,$ $\neg c$ | $\neg a, \neg b, \neg d,$ $\neg c, \neg e$ |
| Cl. 1: $a, \neg b, c$ | $a, \neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $a, c, \neg e$ | $a, c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $\neg e$ | ✓ |
| Cl. 4: $b, \neg c, e$ | $b, \neg c, e$ | $b, \neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| Cl. 5: $c, e$ | $c, e$ | $c, e$ | $c, e$ | $c, e$ | $e$ | {} ✗ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $\neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| BCP | - | - | $\neg d$ | - | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | $\neg c$ | - | - |

Conflict in step 6



$$\frac{3.\ a \vee c \vee \neg e \qquad 5.\ c \vee e}{a \vee c}$$

| Step | 7 | 8 | 9 |
|---|---|---|---|
| Decision Level | 2 | 2 | 2 |
| Assignment | $\neg a, \neg b, \neg d$ | $\neg a, \neg b, \neg d,$ $c$ | $\neg a, \neg b, \neg d,$ $c, e$ |
| Cl. 1: $a, \neg b, c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b, \neg c$ | ✓ | ✓ | ✓ |
| Cl. 3: $a, c, \neg e$ | $c, \neg e$ | ✓ | ✓ |
| Cl. 4: $b, \neg c, e$ | $\neg c, e$ | $e$ | ✓ |
| Cl. 5: $c, e$ | $c, e$ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg b, d$ | ✓ | ✓ | ✓ |
| Cl. 8: $\neg c, e$ | $\neg c, e$ | $e$ | ✓ |
| Cl. 9: $a, c$ | $c$ | ✓ | ✓ |
| BCP | $c$ | $e$ | - |
| PL | - | - | - |
| Decision | - | - | SAT |

2.2.72 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, d\}$

Clause 2: $\{a, c\}$

Clause 3: $\{\neg a, b, \neg c\}$

Clause 4: $\{\neg b, e\}$

Clause 5: $\{a, \neg c\}$

Clause 6: $\{\neg a, \neg e, \neg d\}$

Clause 7: $\{a, \neg b\}$

Clause 8: $\{b, \neg d\}$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ |
| Cl. 1: $a, d$ | $a, d$ | $d$ | $d$ | $d$ |
| Cl. 2: $a, c$ | $a, c$ | $c$ | $c$ | $\{\}$ ✗ |
| Cl. 3: $\neg a, b, \neg c$ | $\neg a, b, \neg c$ | ✔ | ✔ | ✔ |
| Cl. 4: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | ✔ | ✔ |
| Cl. 5: $a, \neg c$ | $a, \neg c$ | $\neg c$ | $\neg c$ | ✔ |
| Cl. 6: $\neg a, \neg e, \neg d$ | $\neg a, \neg e, \neg d$ | ✔ | ✔ | ✔ |
| Cl. 7: $a, \neg b$ | $a, \neg b$ | $\neg b$ | ✔ | ✔ |
| Cl. 8: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $\neg d$ | $\neg d$ |
| BCP | - | $\neg b$ | $\neg c$ | - |
| PL | - | - | - | - |
| Decision | $\neg a$ | - | - | - |

Conflict in step 4



$$\frac{2.\ a \vee c \qquad 5.\ a \vee \neg c}{a}$$

| Step | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg c$ | $a, \neg c, \neg d$ | $a, \neg c, \neg d, \neg b$ |
| Cl. 1: $a, d$ | $a, d$ | ✔ | ✔ | ✔ | ✔ |
| Cl. 2: $a, c$ | $a, c$ | ✔ | ✔ | ✔ | ✔ |
| Cl. 3: $\neg a, b, \neg c$ | $\neg a, b, \neg c$ | $b, \neg c$ | ✔ | ✔ | ✔ |
| Cl. 4: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | ✔ |
| Cl. 5: $a, \neg c$ | $a, \neg c$ | ✔ | ✔ | ✔ | ✔ |
| Cl. 6: $\neg a, \neg e, \neg d$ | $\neg a, \neg e, \neg d$ | $\neg e, \neg d$ | $\neg e, \neg d$ | ✔ | ✔ |
| Cl. 7: $a, \neg b$ | $a, \neg b$ | ✔ | ✔ | ✔ | ✔ |
| Cl. 8: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | ✔ | ✔ |
| Cl. 9: $a$ | $a$ | ✔ | ✔ | ✔ | ✔ |
| BCP | $a$ | - | - | - | - |
| PL | - | $\neg c$ | $\neg d$ | $\neg b$ | - |
| Decision | - | - | - | - | SAT |

2.2.73 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, \neg d\}$

Clause 2:  $\{a, b, c\}$

Clause 3:  $\{\neg a, b\}$

Clause 4:  $\{\neg b, c\}$

Clause 5:  $\{\neg c, d\}$
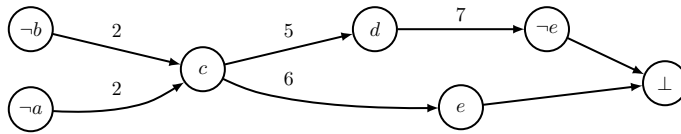
Clause 6:  $\{\neg c, e\}$

Clause 7:  $\{\neg d, \neg e\}$

Solution

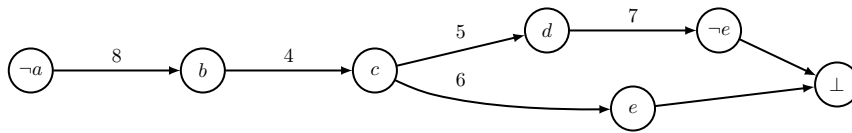| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, d$ | $\neg a, \neg b, c, d, \neg e$ |
| Cl. 1: $\neg a, \neg d$ | $\neg a, \neg d$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, b, c$ | $a, b, c$ | $b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| BCP | - | - | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | - |

Conflict in step 6

$$\frac{\dfrac{7.\ \neg d \vee \neg e \qquad 5.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 6.\ \neg c \vee e}{\dfrac{\neg c \qquad\qquad 2.\ a \vee b \vee c}{a \vee b}}$$

| Step | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 1 | 1 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, c$ | $\neg a, b, c, d$ | $\neg a, b, c, d, \neg e$ |
| Cl. 1: $\neg a, \neg d$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | ✓ | ✓ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ | {} ✗ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg e$ | ✓ |
| Cl. 8: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $c$ | $d$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | - |

Conflict in step 11

$$\frac{\dfrac{\dfrac{7.\ \neg d \vee \neg e \qquad 5.\ \neg c \vee d}{\neg e \vee \neg c} \qquad 6.\ \neg c \vee e}{\neg c \qquad\qquad 4.\ \neg b \vee c}}{\dfrac{\neg b \qquad\qquad 8.\ a \vee b}{a}}$$

| Step | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a,b$ | $a,b,c$ | $a,b,c,$ $\neg d$ |
| Cl. 1: $\neg a, \neg d$ | $\neg a, \neg d$ | $\neg d$ | $\neg d$ | $\neg d$ | ✓ |
| Cl. 2: $a,b,c$ | $a,b,c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, b$ | $\neg a, b$ | $b$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $c$ | ✓ | ✓ |
| Cl. 5: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | {} ✗ |
| Cl. 6: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ |
| Cl. 7: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | ✓ |
| Cl. 8: $a,b$ | $a,b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $b$ | $c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |

Conflict in step 16



$$\frac{\dfrac{\dfrac{\dfrac{\overline{5.\ \neg c \vee d \qquad 4.\ \neg b \vee c}}{d \vee \neg b} \qquad 1.\ \neg a \vee \neg d}{\neg b \vee \neg a} \qquad 3.\ \neg a \vee b}{\neg a} \qquad 9.\ a}{\bot}$$

2.2.74 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, \neg c, \neg e\}$

Clause 2: $\{a, \neg e\}$

Clause 3: $\{b, e\}$

Clause 4: $\{\neg b, d\}$

Clause 5: $\{\neg b, \neg d\}$

Clause 6: $\{c, \neg d\}$

Clause 7: $\{c, d\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, e$ | $a, c, e, \neg b$ |
| Cl. 1: $a, \neg c, \neg e$ | $a, \neg c, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, \neg e$ | $a, \neg e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, e$ | $b, e$ | $b, e$ | $b, e$ | ✓ | ✓ |
| Cl. 4: $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | ✓ |
| Cl. 5: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | ✓ |
| Cl. 6: $c, \neg d$ | $c, \neg d$ | $c, \neg d$ | ✓ | ✓ | ✓ |
| Cl. 7: $c, d$ | $c, d$ | $c, d$ | ✓ | ✓ | ✓ |
| BCP | - | - | - | - | - |
| PL | $a$ | $c$ | $e$ | $\neg b$ | - |
| Decision | - | - | - | - | SAT |

2.2.75 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{b, c, d\}$

Clause 2: $\{\neg a, \neg c\}$

Clause 3: $\{b, c\}$

Clause 4: $\{\neg b, \neg d\}$

Clause 5: $\{\neg d, e\}$

Clause 6: $\{d, e\}$

Clause 7: $\{\neg c, \neg e\}$

Clause 8: $\{\neg b, c, \neg d\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c,$ $\neg e$ | $\neg a, \neg b, c,$ $\neg e, \neg d$ |
| Cl. 1: $b, c, d$ | $b, c, d$ | $b, c, d$ | $c, d$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg c$ | $\neg a, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, c$ | $b, c$ | $b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | $\neg d$ | ✓ |
| Cl. 6: $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d, e$ | $d$ | {} ✗ |
| Cl. 7: $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg e$ | ✓ | ✓ |
| Cl. 8: $\neg b, c, d$ | $\neg b, c, d$ | $\neg b, c, d$ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | - | $c$ | $\neg e$ | $\neg d$ | - |
| PL | $\neg a$ | - | - | - | - | - |
| Decision | - | $\neg b$ | - | - | - | - |

Conflict in step 6



$$\frac{\dfrac{5.\ \neg d \vee e \qquad 6.\ d \vee e}{e} \qquad 7.\ \neg c \vee \neg e}{\dfrac{\neg c \qquad\qquad\qquad 3.\ b \vee c}{b}}$$

| Step | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, \neg d$ | $\neg a, b, \neg d,$ $c$ | $\neg a, b, \neg d,$ $c, \neg e$ |
| Cl. 1: $b, c, d$ | $b, c, d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg d$ | $\neg b, \neg d$ | $\neg d$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg d, e$ | $\neg d, e$ | $\neg d, e$ | ✓ | ✓ | ✓ |
| Cl. 6: $d, e$ | $d, e$ | $d, e$ | $e$ | $e$ | {} ✗ |
| Cl. 7: $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg c, \neg e$ | $\neg e$ | ✓ |
| Cl. 8: $\neg b, c, d$ | $\neg b, c, d$ | $c, d$ | $c$ | ✓ | ✓ |
| Cl. 9: $b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $\neg d$ | $c$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |

Conflict in step 11



$$\frac{\dfrac{\dfrac{\dfrac{7.\ \neg c \vee \neg e \qquad 6.\ d \vee e}{\neg c \vee d} \qquad 8.\ \neg b \vee c \vee d}{d \vee \neg b} \qquad 4.\ \neg b \vee \neg d}{\neg b} \qquad 9.\ b}{\bot}$$

2.2.76  Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable.  Decide variables in alphabetical order starting with the

*positive* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, d\}$

Clause 2: $\{\neg d, c, \neg a\}$

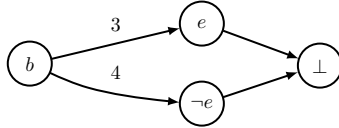Clause 3: $\{\neg b, e\}$

Clause 4: $\{\neg b, \neg e\}$

Clause 5: $\{\neg b, f\}$

Clause 6: $\{b, \neg f\}$

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**Solution**

| Step | 1 | 2 | 3 | 4 |
|------|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, e$ |
| Cl. 1: $\neg a, d$ | $\neg a, d$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg d, c, \neg a$ | $\neg d, c, \neg a$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $e$ | ✓ |
| Cl. 4: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg e$ | $\{\}$ ✗ |
| Cl. 5: $\neg b, f$ | $\neg b, f$ | $\neg b, f$ | $f$ | $f$ |
| Cl. 6: $b, \neg f$ | $b, \neg f$ | $b, \neg f$ | ✓ | ✓ |
| BCP | - | - | $e$ | - |
| PL | $\neg a$ | - | - | - |
| Decision | - | $b$ | - | - |

Conflict in step 4



$$\frac{3.\ \neg b \vee e \qquad 4.\ \neg b \vee \neg e}{\neg b}$$

| Step | 5 | 6 | 7 |
|------|---|---|---|
| Decision Level | 0 | 0 | 0 |
| Assignment | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg f$ |
| Cl. 1: $\neg a, d$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg d, c, \neg a$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, e$ | $\neg b, e$ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg e$ | $\neg b, \neg e$ | ✓ | ✓ |
| Cl. 5: $\neg b, f$ | $\neg b, f$ | ✓ | ✓ |
| Cl. 6: $b, \neg f$ | $b, \neg f$ | $\neg f$ | ✓ |
| Cl. 7: $\neg b$ | $\neg b$ | ✓ | ✓ |
| BCP | $\neg b$ | $\neg f$ | - |
| PL | - | - | - |
| Decision | - | - | SAT |

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**2.2.77** Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in

UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, b, c\}$

Clause 2: $\{a, c\}$

Clause 3: $\{b, \neg c\}$

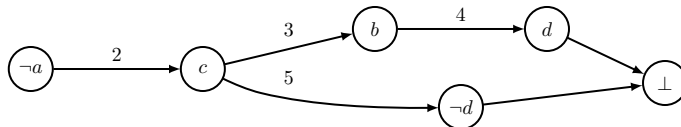Clause 4: $\{\neg b, d\}$

Clause 5: $\{\neg c, \neg d\}$

Clause 6: $\{c, e\}$

Clause 7: $\{c, \neg e\}$

Solution

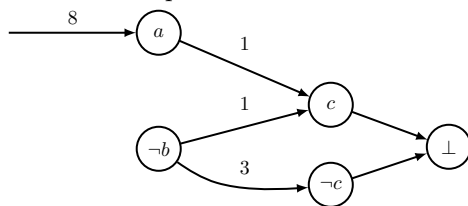| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, c$ | $\neg a, c, b$ | $\neg a, c, b, \neg d$ |
| Cl. 1: $\neg a, b, c$ | $\neg a, b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, c$ | $a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $b$ | ✓ | ✓ |
| Cl. 4: $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | $d$ | {} ✗ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg d$ | $\neg d$ | ✓ |
| Cl. 6: $c, e$ | $c, e$ | $c, e$ | ✓ | ✓ | ✓ |
| Cl. 7: $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | ✓ | ✓ | ✓ |
| BCP | - | $c$ | $b$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |

Conflict in step 5



$$\frac{\dfrac{4.\ \neg b \vee d \qquad 3.\ b \vee \neg c}{d \vee \neg c} \qquad 5.\ \neg c \vee \neg d}{\dfrac{\neg c}{a} \qquad 2.\ a \vee c}$$

| Step | 6 | 7 | 8 | 9 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 |
| Assignment | - | $a$ | $a, \neg b$ | $a, \neg b, \neg c$ |
| Cl. 1: $\neg a, b, c$ | $\neg a, b, c$ | $b, c$ | $c$ | {} ✗ |
| Cl. 2: $a, c$ | $a, c$ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | $b, \neg c$ | $\neg c$ | ✓ |
| Cl. 4: $\neg b, d$ | $\neg b, d$ | $\neg b, d$ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | ✓ |
| Cl. 6: $c, e$ | $c, e$ | $c, e$ | $c, e$ | $e$ |
| Cl. 7: $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $\neg e$ |
| Cl. 8: $a$ | $a$ | ✓ | ✓ | ✓ |
| BCP | $a$ | - | $\neg c$ | - |
| PL | - | - | - | - |
| Decision | - | $\neg b$ | - | - |

Conflict in step 9



$$\frac{\dfrac{1.\ \neg a \vee b \vee c \qquad 8.\ a}{b \vee c} \qquad 3.\ b \vee \neg c}{b}$$

| Step | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | $a$ | $a,b$ | $a,b,d$ | $a,b,d,$ $\neg c$ | $a,b,d,$ $\neg c, \neg e$ |
| Cl. 1: $\neg a, b, c$ | $b,c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $a, c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $b, \neg c$ | $b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, d$ | $\neg b, d$ | $d$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c$ | ✓ | ✓ |
| Cl. 6: $c, e$ | $c, e$ | $c, e$ | $c, e$ | $e$ | {} ✗ |
| Cl. 7: $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $\neg e$ | ✓ |
| Cl. 8: $a$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | $d$ | $\neg c$ | $\neg e$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | UNSAT |

Conflict in step 14



2.2.78 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{b, \neg d, a\}$

Clause 2: $\{\neg a, c\}$

Clause 3: $\{\neg a, b, c\}$

Clause 4: $\{\neg b, e\}$

Clause 5: $\{a, d\}$

Clause 6: $\{a, \neg c\}$

Clause 7: $\{\neg a, \neg e, \neg d\}$

Clause 8: $\{a, \neg b\}$

**Solution**

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c, \neg d$ |
| Cl. 1: $b, \neg d, a$ | $b, \neg d, a$ | $b, \neg d$ | $\neg d$ | $\neg d$ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, b, c$ | $\neg a, b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $a, d$ | $a, d$ | $d$ | $d$ | $d$ | {} ✗ |
| Cl. 6: $a, \neg c$ | $a, \neg c$ | $\neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 7: $\neg a, \neg e, \neg d$ | $\neg a, \neg e, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 8: $a, \neg b$ | $a, \neg b$ | $\neg b$ | ✓ | ✓ | ✓ |
| BCP | - | $\neg b$ | $\neg c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |

Conflict in step 5



$$\cfrac{\cfrac{1.\ b \vee \neg d \vee a \qquad 5.\ a \vee d}{b \vee a} \qquad 8.\ a \vee \neg b}{a}$$

| Step | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, \neg b$ | $a, c, \neg b, \neg d$ |
| Cl. 1: $b, \neg d, a$ | $b, \neg d, a$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg a, b, c$ | $\neg a, b, c$ | $b, c$ | ✓ | ✓ | ✓ |
| Cl. 4: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | ✓ | ✓ |
| Cl. 5: $a, d$ | $a, d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $a, \neg c$ | $a, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg a, \neg e, \neg d$ | $\neg a, \neg e, \neg d$ | $\neg e, \neg d$ | $\neg e, \neg d$ | $\neg e, \neg d$ | ✓ |
| Cl. 8: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 9: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | - | - | - |
| PL | - | - | $\neg b$ | $\neg d$ | - |
| Decision | - | - | - | - | SAT |

2.2.79 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b, c\}$

Clause 2: $\{\neg a, b\}$

Clause 3: $\{\neg b, c\}$

Clause 4: $\{c, d\}$

Clause 5: $\{\neg c, e\}$

Clause 6: $\{\neg d, \neg e\}$

| Solution | | | | | | |
|---|---|---|---|---|---|---|
| Step | 1 | 2 | 3 | 4 | 5 | 6 |
| Decision Level | 0 | 1 | 2 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, e$ | $\neg a, \neg b, c, e, \neg d$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | $b, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, b$ | $\neg a, b$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $c, d$ | $c, d$ | $c, d$ | $c, d$ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | ✓ | ✓ |
| Cl. 6: $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d, \neg e$ | $\neg d$ | ✓ |
| BCP | - | - | $c$ | $e$ | $\neg d$ | - |
| PL | - | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - | SAT |

2.2.80 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *positive* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{\neg a, d\}$

Clause 2: $\{\neg d, c, \neg a\}$

Clause 3: $\{\neg b, e\}$

Clause 4: $\{\neg b, \neg e\}$

Clause 5: $\{b, f\}$

Clause 6: $\{b, \neg f\}$

Solution

| Step | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, b$ | $\neg a, b, e$ |
| Cl. 1: $\neg a, d$ | $\neg a, d$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg d, c, \neg a$ | $\neg d, c, \neg a$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, e$ | $\neg b, e$ | $\neg b, e$ | $e$ | ✓ |
| Cl. 4: $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg b, \neg e$ | $\neg e$ | {} ✗ |
| Cl. 5: $b, f$ | $b, f$ | $b, f$ | ✓ | ✓ |
| Cl. 6: $b, \neg f$ | $b, \neg f$ | $b, \neg f$ | ✓ | ✓ |
| BCP | - | - | $e$ | - |
| PL | $\neg a$ | - | - | - |
| Decision | - | $b$ | - | - |

Conflict in step 4



$$\frac{3.\ \neg b \vee e \qquad 4.\ \neg b \vee \neg e}{\neg b}$$

| Step | 5 | 6 | 7 |
|---|---|---|---|
| Decision Level | 0 | 0 | 0 |
| Assignment | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, f$ |
| Cl. 1: $\neg a, d$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg d, c, \neg a$ | ✓ | ✓ | ✓ |
| Cl. 3: $\neg b, e$ | $\neg b, e$ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg e$ | $\neg b, \neg e$ | ✓ | ✓ |
| Cl. 5: $b, f$ | $b, f$ | $f$ | ✓ |
| Cl. 6: $b, \neg f$ | $b, \neg f$ | $\neg f$ | {} ✗ |
| Cl. 7: $\neg b$ | $\neg b$ | ✓ | ✓ |
| BCP | $\neg b$ | $f$ | - |
| PL | - | - | - |
| Decision | - | - | UNSAT |

Conflict in step 7



$$\frac{\dfrac{5.\ b \vee f \qquad 6.\ b \vee \neg f}{b} \qquad 7.\ \neg b}{\bot}$$

2.2.81 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, \neg b, c\}$

Clause 2: $\{b, \neg c, d\}$

Clause 3: $\{a, \neg b\}$

Clause 4: $\{a, c\}$

Clause 5: $\{\neg c, \neg d\}$

Clause 6: $\{\neg a, c\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 1 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, \neg d$ |
| Cl. 1: $a, \neg b, c$ | $a, \neg b, c$ | $\neg b, c$ | ✓ | ✓ | ✓ |
| Cl. 2: $b, \neg c, d$ | $b, \neg c, d$ | $b, \neg c, d$ | $\neg c, d$ | $d$ | $\{\}$ ✗ |
| Cl. 3: $a, \neg b$ | $a, \neg b$ | $\neg b$ | ✓ | ✓ | ✓ |
| Cl. 4: $a, c$ | $a, c$ | $c$ | $c$ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg d$ | ✓ |
| Cl. 6: $\neg a, c$ | $\neg a, c$ | ✓ | ✓ | ✓ | ✓ |
| BCP | - | $\neg b$ | $c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | - | - | - | - |

Conflict in step 5



$$\underline{2.\ b \lor \neg c \lor d \qquad 5.\ \neg c \lor \neg d}$$
$$\underline{b \lor \neg c \qquad\qquad\qquad 3.\ a \lor \neg b}$$
$$\underline{\neg c \lor a \qquad\qquad\qquad 4.\ a \lor c}$$
$$a$$

| Step | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, c$ | $a, c, \neg d$ | $a, c, \neg d, b$ |
| Cl. 1: $a, \neg b, c$ | $a, \neg b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $b, \neg c, d$ | $b, \neg c, d$ | $b, \neg c, d$ | $b, d$ | $b$ | ✓ |
| Cl. 3: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 4: $a, c$ | $a, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 5: $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg c, \neg d$ | $\neg d$ | ✓ | ✓ |
| Cl. 6: $\neg a, c$ | $\neg a, c$ | $c$ | ✓ | ✓ | ✓ |
| Cl. 7: $a$ | $a$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $a$ | $c$ | $\neg d$ | $b$ | - |
| PL | - | - | - | - | - |
| Decision | - | - | - | - | SAT |

2.2.82 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the learned clause to the table.

If the set of clauses resulted in SAT, give a satisfying model. If the set of clauses resulted in UNSAT, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b, c\}$

Clause 2: $\{\neg b, \neg c, e\}$

Clause 3: $\{b, e\}$

Clause 4: $\{b, \neg d\}$

Clause 5: $\{\neg c, d\}$

Clause 6: $\{\neg c, e\}$

Clause 7: $\{\neg a, \neg b, \neg c\}$

Clause 8: $\{a, c, \neg e\}$

Solution

| Step | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Decision Level | 0 | 1 | 2 | 2 | 2 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ | $\neg a, \neg b, c$ | $\neg a, \neg b, c, \neg d$ |
| Cl. 1: $a, b, c$ | $a, b, c$ | $b, c$ | $c$ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b, \neg c$ | $\neg a, \neg b, \neg c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $a, c, \neg e$ | $a, c, \neg e$ | $c, \neg e$ | $c, \neg e$ | ✓ | ✓ |
| Cl. 4: $\neg b, \neg c, e$ | $\neg b, \neg c, e$ | $\neg b, \neg c, e$ | ✓ | ✓ | ✓ |
| Cl. 5: $b, e$ | $b, e$ | $b, e$ | $e$ | $e$ | $e$ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | $b, \neg d$ | $\neg d$ | $\neg d$ | ✓ |
| Cl. 7: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | $d$ | $\{\}$ ✗ |
| Cl. 8: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $e$ | $e$ |
| BCP | - | - | $c$ | $\neg d$ | - |
| PL | - | - | - | - | - |
| Decision | $\neg a$ | $\neg b$ | - | - | - |

Conflict in step 5



$$\dfrac{\dfrac{7.\ \neg c \vee d \qquad 1.\ a \vee b \vee c}{d \vee a \vee b} \qquad 6.\ b \vee \neg d}{a \vee b}$$

| Step | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| Decision Level | 1 | 1 | 1 | 2 | 2 |
| Assignment | $\neg a$ | $\neg a, b$ | $\neg a, b, d$ | $\neg a, b, d, \neg c$ | $\neg a, b, d, \neg c, \neg e$ |
| Cl. 1: $a, b, c$ | $b, c$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg a, \neg b, \neg c$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Cl. 3: $a, c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $c, \neg e$ | $\neg e$ | ✓ |
| Cl. 4: $\neg b, \neg c, e$ | $\neg b, \neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| Cl. 5: $b, e$ | $b, e$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 6: $b, \neg d$ | $b, \neg d$ | ✓ | ✓ | ✓ | ✓ |
| Cl. 7: $\neg c, d$ | $\neg c, d$ | $\neg c, d$ | ✓ | ✓ | ✓ |
| Cl. 8: $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | $\neg c, e$ | ✓ | ✓ |
| Cl. 9: $a, b$ | $b$ | ✓ | ✓ | ✓ | ✓ |
| BCP | $b$ | - | - | $\neg e$ | - |
| PL | - | $d$ | - | - | - |
| Decision | - | - | $\neg c$ | - | SAT |

2.2.83 Use the DPLL algorithm with conflict-driven clause learning to determine whether or not the set of clauses given is satisfiable. Decide variables in alphabetical order starting with the *negative* phase. For conflicts, draw conflict graphs after the end of the table, and add the

learned clause to the table.

If the set of clauses resulted in `SAT`, give a satisfying model. If the set of clauses resulted in `UNSAT`, give a resolution proof that shows that the conjunction of the clauses from the table is unsatisfiable.

Clause 1: $\{a, b\}$

Clause 2: $\{\neg b, c\}$

Clause 3: $\{\neg a, \neg c\}$

Clause 4: $\{b, c\}$

Clause 5: $\{a, \neg b\}$

Clause 6: $\{\neg b, \neg c\}$

---

**Solution**

| Step | 1 | 2 | 3 |
|---|---|---|---|
| Decision Level | 0 | 1 | 1 |
| Assignment | - | $\neg a$ | $\neg a, \neg b$ |
| Cl. 1: $a, b$ | $a, b$ | $b$ | $\{\}$ ✗ |
| Cl. 2: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | ✓ |
| Cl. 3: $\neg a, \neg c$ | $\neg a, \neg c$ | ✓ | ✓ |
| Cl. 4: $b, c$ | $b, c$ | $b, c$ | $c$ |
| Cl. 5: $a, \neg b$ | $a, \neg b$ | $\neg b$ | ✓ |
| Cl. 6: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | ✓ |
| BCP | - | $\neg b$ | - |
| PL | - | - | - |
| Decision | $\neg a$ | - | - |

Conflict in step 3
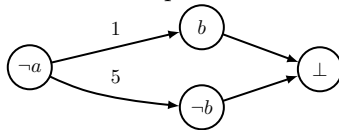


$$\frac{1.\ a \vee b \qquad 5.\ a \vee \neg b}{a}$$

| Step | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| Decision Level | 0 | 0 | 0 | 0 |
| Assignment | - | $a$ | $a, \neg c$ | $a, \neg c, \neg b$ |
| Cl. 1: $a, b$ | $a, b$ | ✓ | ✓ | ✓ |
| Cl. 2: $\neg b, c$ | $\neg b, c$ | $\neg b, c$ | $\neg b$ | ✓ |
| Cl. 3: $\neg a, \neg c$ | $\neg a, \neg c$ | $\neg c$ | ✓ | ✓ |
| Cl. 4: $b, c$ | $b, c$ | $b, c$ | $b$ | $\{\}$ ✗ |
| Cl. 5: $a, \neg b$ | $a, \neg b$ | ✓ | ✓ | ✓ |
| Cl. 6: $\neg b, \neg c$ | $\neg b, \neg c$ | $\neg b, \neg c$ | ✓ | ✓ |
| Cl. 7: $a$ | $a$ | ✓ | ✓ | ✓ |
| BCP | $a$ | $\neg c$ | $\neg b$ | - |
| PL | - | - | - | - |
| Decision | - | - | - | UNSAT |

Conflict in step 7



$$\cfrac{\cfrac{\cfrac{2.\ \neg b \vee c \qquad 4.\ b \vee c}{c} \qquad 3.\ \neg a \vee \neg c}{\neg a} \qquad 7.\ a}{\bot}$$