Efficient Rewriting of XPath Queries Using Query Set Specifications

Bogdan Cautis
Telecom ParisTech
cautis@telecom-paristech.fr

Alin Deutsch
UC San Diego
deutsch@cs.ucsd.edu

Nicola Onose
UC San Diego
nicola@cs.ucsd.edu

Vasilis Vassalos
Athens Univ. of Economics and Business
vassalos@aueb.gr

ABSTRACT

We study the problem of querying XML data sources that accept only a limited set of queries, such as sources accessible by Web services which can implement very large (potentially infinite) families of XPath queries. To compactly specify such families of queries we adopt the Query Set Specifications [14], a formalism close to context-free grammars.

We say that query Q is *expressible* by the specification $\mathcal{P}$ if it is equivalent to some expansion of $\mathcal{P}$. Q is *supported* by $\mathcal{P}$ if it has an equivalent rewriting using some finite set of $\mathcal{P}$'s expansions. We study the complexity of expressibility and support and identify large classes of XPath queries for which there are efficient (PTIME) algorithms. Our study considers both the case in which the XML nodes in the results of the queries lose their original identity and the one in which the source exposes persistent node ids.

1. INTRODUCTION

Current Web data sources usually do not allow clients to ask arbitrary queries, but instead publish as Web Services a set of queries they are willing to answer, which we will refer to as *views* . Main reasons for that are performance requirements, business model considerations and access restrictions deriving from security policies. Querying such sources involves finding one or several legal views that can be used to answer the client query.

Of particular interest is the case when the set of views is very large (possibly exponential in the size of the schema or even infinite), precluding explicit enumeration by the source owner as well as full comprehension by the client query developer. In such scenarios, recent proposals advocate the owner's specifying the set of legal views implicitly, using a compact representation (in the same spirit in which a potentially infinite language is finitely specified by a grammar). Clients are unaware of the legal views, and simply pose their query against a logical schema exported by the source (the same schema against which the views are defined). While this approach provides a simpler interface to source owner and client, it raises a technical challenge, as now the system has to automatically identify and extract from the compact encoding a finite set of legal views that can be used to answer the client query.

This problem has been the object of several recent studies in a relational setting [9, 16, 6], but has not been addressed for sources that publish XML data (as is the case for most current Web Services). Since our focus is on practical algorithms, we consider sources that make XML data available through sets of views belonging to an XPath fragment for which the basic building blocks of rewriting algorithms, namely containment and equivalence, are tractable [11]. As a formalism for compactly representing large sets of such views, we adopt a variation of the Query Set Specification Language(QSSL) [14], a grammar-like formalism for specifying XPath view families (see also [12]).

Expressibility and support. As in the literature on sources exporting sets of legal relational queries [16, 6], we consider two settings for query answering. The first one is when the client query has to be fully answered by asking one legal query over the source, with no post-processing of its result. The corresponding decision problem is called *expressibility* [6]: we say that query q is *expressible* if it is equivalent to a view published by the source. The second setting is when the capabilities of the source are extended by a *source wrapper* [13] that intercepts the client query, finds an equivalent rewriting for it in terms of the views, post-processes the results locally and returns the query result to the client. The associated problem is called *support* [6]: given a rewriting query language $\mathcal{L}_R$, q is *supported* by $\mathcal{P}$ if it has an equivalent rewriting in $\mathcal{L}_R$ using some finite set of legal queries supported by the source.

Expressibility and support generalize the problems of equivalence and existence of a rewriting using views from the classical case in which the set of views is explicitly listed to the case in which this set is very large, potentially infinite, being specified implicitly by a compact representation.

XPath rewriting. Earlier research [17, 10] on XPath rewriting studied the problem of equivalently rewriting an XPath by navigating inside a *single* materialized XPath view. This is the only kind of rewritings supported when the query cache can only store or can only obtain *copies* of the XML elements in the query answer, and so the original node identities are lost.

We have recently witnessed an industrial trend towards enhancing XPath queries with the ability to expose node identifiers and exploit them using intersection of node sets (via identity-based equality). This trend is supported by systems such as [3]. This development enables for the first time multiple-view rewritings obtained by intersecting several materialized view results. The single-view rewritings considered in early XPath research have only limited benefit, as many queries with no single-view rewriting can be rewritten using multiple views. In this paper, we consider both the case in which the XML nodes in the results of the queries lose their original identity (hence a rewriting can only use one view) and the one in which the source exposes persistent node ids (and rewritings using multiple views are possible).

EXAMPLE 1.1. *Throughout this paper we consider the example*

VLDB '09, August 24-28, 2009, Lyon, France

of a tourism agency that allows to find organized trips matching user criteria. The set of allowed queries is specified by a compact QSS encoding (to be described shortly). On the schema of views published by the source, the client formulates a query q_1, asking for museums during a tour in whose schedule there is also a slot for taking a walk and which is part of a guided secondary trip:

q_1*: doc(T)//vacation//trip/trip[guide]//tour[schedule//walk]/museum*

The system analyzes the query and the specification and finds two views that may be relevant for answering q_1. These are v_1, which returns museums in secondary trips for which there is a guide:

v_1*: doc(T)//vacation//trip/trip[guide]//museum*

and v_2, which returns museums on a tour in which there has been also scheduled a walk:

v_2*: doc(T)//vacation//trip//tour[schedule//walk]/museum*

q_1 cannot be answered just by navigating into the result of v_1 or into the result of v_2. The reason is that q_1 needs both to enforce that the trip has a guide and that the tour has a walk in the schedule. v_1 or v_2 taken individually can enforce one of the two conditions, but not both, and navigation down into the view does not help either, since the output node museum is below the trip and tour nodes. Since no other views published by the source can contribute to rewriting q_1, in the absence of ids, the system will reject q_1, as it is neither expressed, nor supported by the source.

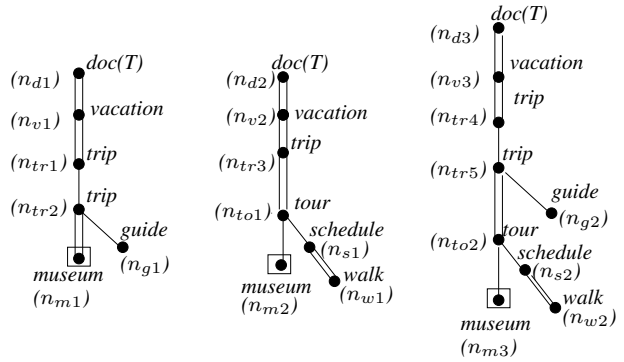
However, if the views expose persistent node ids, we will show that q_1 can be rewritten as an intersection of v_1 and v_2.

Contributions. We study the complexity of expressibility and support and identify large classes of XPath queries for which there are efficient (PTIME) algorithms. For expressibility, we give a PTIME decision procedure that works for any QSS and for any XPath query from a large fragment allowing child and descendant navigation and predicates. We show that support in the absence of ids remains in PTIME, for the same XPath fragment for which we studied expressibility. However, for this fragment, support in the presence of ids becomes coNP-hard. This is a consequence of previous results [5], showing that rewriting XPath using an intersection of XPath views (a problem subsumed by support) is already coNP-hard. This is a major difference with respect to the relational case, in which support and expressibility were proven inter-reducible [6]. Since our focus is on practical algorithms, we propose a PTIME algorithm for id-based support that is sound for any XPath query, and becomes complete under fairly permissive restrictions on the query, without further restricting the language of the views. Our results are in stark contrast with previous results in the relational setting [9, 16], where already the simple language of conjunctive queries leads to EXPTIME completeness of expressivity and support [6], but on the other hand is closed under intersection, which poses no additional problem.

Outline of the paper. The paper is structured as follows. Section 2 presents the language of client queries (tree patterns) and the language of query rewriting plans (tree patterns and intersections thereof). Section 3 describes the query set specifications (QSS). The problem of expressibility is analyzed in Section 4. The problem of support is studied starting from Section 5, first in the absence of persistent ids and then in their presence (Sections 6, 7, 8). QSS and rewriting language extensions are presented in Sections 9, 10. Section 11 discussed related work and Section 12 concludes.

2. XPATH AND TREE PATTERNS

We consider an XML document as an unranked, unordered rooted tree t modeled by a set of edges $\text{EDGES}(t)$, a set of nodes $\text{NODES}(t)$, a distinguished root node $\text{ROOT}(t)$ and a labeling function λ_t, assigning to each node a label from an infinite alphabet Σ.



view v_1 view v_2 query q_1

Figure 1: The tree patterns of queries v_1, v_2 and q_1

We consider XPath queries with child / and descendant // navigation, without wildcards. We call the resulting language *XP*, and define its grammar as:

$$\begin{array}{rcl} apath & ::= & doc(\text{“}name\text{”})/rpath \mid doc(\text{“}name\text{”})//rpath \\ rpath & ::= & step \mid rpath/rpath \mid rpath//rpath \\ step & ::= & label\ pred \\ pred & ::= & \epsilon \mid [rpath] \mid [.//rpath] \mid pred\ pred \end{array}$$

The sub-expressions inside brackets are called *predicates*.

All definitions and results of this paper extend naturally when allowing equality with constants in predicates. For presentation simplicity, this feature will be ignored in the core of the paper, and is briefly discussed in Section 9.

In the following, we will prefer an alternative representation for XML queries widely used in literature, the one of *tree patterns* [11]:

DEFINITION 2.1. *A* tree pattern *p is a non empty rooted tree, with a set of nodes $\text{NODES}(p)$ labeled with symbols from Σ, a distinguished node called the* output node $\text{OUT}(p)$*, and two types of edges:* child edges*, labeled by / and* descendant edges*, labeled by //. The root of p is denoted $\text{ROOT}(p)$.*

Any *XP* expression can be translated into a tree pattern query and vice versa (see, for instance [11]). For a given tree pattern query p, *xpath*(p) is the associated *XP* expression.

EXAMPLE 2.1. *Figure 1 shows the tree patterns corresponding to v_1, v_2 and q_1 from Example 1.1. Each node has a label and a unique node symbol, written inside parenthesis. Output nodes are distinguished in the graphical representation by a square.*

The semantics of a tree pattern can be given using embeddings:

DEFINITION 2.2. *An* embedding *of a tree pattern p into a tree t over Σ is a function e from $\text{NODES}(p)$ to $\text{NODES}(t)$ that has the following properties: (1) $e(\text{ROOT}(p)) = \text{ROOT}(t)$; (2) for any $n \in \text{NODES}(p)$, $\text{LABEL}(e(n)) = \text{LABEL}(n)$; (3) for any /-edge (n_1, n_2) in p, $(e(n_1), e(n_2))$ is an edge in t; (4) for any //-edge (n_1, n_2) in p, there is a path from $e(n_1)$ to $e(n_2)$ in t.*

The *result* of applying a tree pattern p to an XML tree t is the set: $\{e(\text{OUT}(p)) |$ *e is an embedding of p into t* $\}$

DEFINITION 2.3. *A tree pattern p_1 is* contained *in a tree pattern p_2 iff for any input tree t, $p_1(t) \subseteq p_2(t)$. We write this shortly as $p_1 \sqsubseteq p_2$. We say that p_1 is* equivalent *to p_2, and write $p_1 \equiv p_2$, iff $p_1(t) = p_2(t)$ for any input tree t.*

The same notions are also used on *XP* expressions. A pattern p is said *minimal* [1] if no pattern $p' \equiv p$ can have fewer nodes than p.

DEFINITION 2.4. *A* mapping *between two tree patterns p_1, p_2 is a function $h : \text{NODES}(p_1) \rightarrow \text{NODES}(p_2)$ satisfying properties (2),(4) of an embedding (allowing the target to be a pattern) plus three others: (5) for any $n \in \text{MBN}(p_1)$, $h(n) \in \text{MBN}(p_2)$; (6) for any /-edge (n_1, n_2) in p_1, $(e(n_1), e(n_2))$ is a /-edge in p_2.*

A root-mapping *is a mapping that satisfies (1). An* output-mapping *is a mapping h such that $h(\text{OUT}(p_1)) = \text{OUT}(p_2)$. A* containment mapping *denotes a mapping that is simultaneously a root-mapping and an output-mapping.*

Previous studies [1, 11] show that for two tree patterns p_1 and p_2, $p_2 \sqsubseteq p_1$ iff there is a containment mapping from p_1 into p_2.

For a tree pattern p, we refer to the path starting with $\text{ROOT}(d)$ and ending with $\text{OUT}(p)$ as the *main branch* of p. We refer to the set of nodes on this path as $\text{MBN}(p)$. We say that a pattern is *linear* if it has no side branches. By $\text{MB}(p)$ we denote the linear pattern that is isomorphic with the main branch of p. We call *predicate subtree* of a pattern p any subtree rooted at a non-main branch node.

Intersection. We consider in this paper the extension *XP*$^{\cap}$ of *XP* with respect to intersection, having a straightforward semantics. Its grammar is obtained from that of *XP* by adding the following rule:

$$ipath ::= apath \mid apath \cap ipath$$

By *XP*$^{\cap}$ expressions over a set of documents D we denote those that use only *apath* expressions that navigate inside D's documents.

As in [4], a *code* is a string of symbols from Σ, alternating with either / or //.

DEFINITION 2.5 (INTERLEAVING). *A* interleaving *of a finite set of tree patterns $\mathcal{S}$ is any tree pattern p_i produced as follows:*

1. *let $M = \cup_{p \in S} \text{MBN}(p)$,*
2. *choose a code i and a total onto function f_i that maps M into Σ-positions of i such that:*
   - *(a) for any $n \in M, \text{LABEL}(f_i(n)) = \text{LABEL}(n)$*
   - *(b) for any $p \in \mathcal{S}$, $f_i(\text{ROOT}(p))$ is the first symbol of i,*
   - *(c) for any $p \in \mathcal{S}$, $f_i(\text{OUT}(p))$ is the last symbol of i,*
   - *(d) for any /-edge (n_1, n_2) of any $p \in \mathcal{S}$, i is of the form $\ldots f_i(n_1)/f_i(n_2)\ldots$,*
   - *(e) for any //-edge (n_1, n_2) of any $p \in \mathcal{S}$, i is of the form $\ldots f_i(n_1)\ldots f_i(n_2)\ldots$.*
3. *build the smallest pattern p_i such that:*
   - *(a) i is a code for the main branch of p_i,*
   - *(b) for any $n \in M$ and its image n' in p_i (via f_i), if a predicate subtree st appears below n then a copy of st appears below n', connected by the same kind of edge.*

Two nodes n_1, n_2 from M are said to be collapsed *if $f_i(n_1) = f_i(n_2)$, with f_i as above. The tree patterns p_i thus obtained are called* interleavings *of $\mathcal{S}$ and we denote their set by* interleave($\mathcal{S}$).

EXAMPLE 2.2. *One of the interleavings of v_1 and v_2 from Figure 1 is q_1, as v_1 has a //-edge between nodes n_{tr2} and n_{m1}, which allows the tour from v_2 to appear as a direct parent of museum.*

Considering also unions of tree patterns, having straightforward semantics, one can prove the following intersection-union duality:

LEMMA 2.1. *For any set of XP queries $\mathcal{S} = \{q_1, \ldots, q_n\}$, the XP*$^{\cap}$ *expression $\cap_i q_i$ is equivalent to the union $\cup interleave(\mathcal{S})$.*

The following also holds:

LEMMA 2.2. *A tree pattern is contained in a union of tree patterns iff it is contained in a member of the union. A tree pattern contains a union of patterns iff it contains each member of the union.*

The set of interleavings of a set of patterns $\mathcal{S}$ may be exponentially larger than $\mathcal{S}$. Indeed, it was shown that the *XP*$^{\cap}$ fragment is not included in *XP* (i.e, the union of its interleavings cannot always be reduced to one *XP* query by eliminating the redundant interleavings contained in others) and that an intersection may only be translatable into a union of exponentially many tree patterns (see [4]).

View-based rewriting. Given a set of views $\mathcal{V}$, defined by *XP* queries over a document D, by $D_{\mathcal{V}}$ we denote the set of view documents $\{doc(v) | v \in \mathcal{V}\}$, in which the topmost element is labeled with the view name. Given a query r, expressed in a rewrite language $\mathcal{L}_R$ (e.g., *XP* or *XP*$^{\cap}$), over the view documents $D_{\mathcal{V}}$, we define *unfold*(r) as the $\mathcal{L}_R$ query obtained by replacing in r each $doc(v)/v$ with the definition of v.

Given an *XP* query q and a finite set of *XP* views $\mathcal{V}$ over D, we look for an alternative plan r in $\mathcal{L}_R$, called a *rewriting*, that can be used to answer q. We define rewritings as follows:

DEFINITION 2.6. *For a given document D, an XP query q and XP views $\mathcal{V}$ over D, a* rewrite plan *of q using $\mathcal{V}$ is a query $r \in \mathcal{L}_R$ over $D_{\mathcal{V}}$. If unfold$(r) \equiv q$, then we also say r is a* rewriting *for q.*

According to the definition above, a rewrite plan r in *XP* is of the form $doc(v_j)/v_j$, $doc(v_j)/v_j/p$ or $doc(v_j)/v_j//p$.

Similarly, according to the definition of *XP*$^{\cap}$, a rewrite plan r in *XP*$^{\cap}$ is of the form $r = (\bigcap_{i,j} u_{ij})$, for each u_{ij} being of the form $doc(v_j)/v_j$, $doc(v_j)/v_j/p_i$ or $doc(v_j)/v_j//p_i$. Note that such a query r is a rewriting (i.e., equivalent to q) iff

- each query *unfold*(u_{ij}) contains q, and
- by Lemmas 2.1 and 2.2, q contains all the tree patterns (interleavings) in $interleave(\{\textit{unfold}(u_{ij})\})$.

Further notation. We introduce now some additional notation, which will be first used in Section 7 and can be skipped until then.

A */-pattern* is a tree pattern having only /-edges in the main branch. A */-predicate* (resp. *//-predicate*) is a predicate subtree that is connected by a /-edge (resp. //-edge) to the main branch.

We will refer to main branch nodes of a pattern p by their *rank* in the main branch, i.e. a value in the range 1 to $|\text{MB}(p)|$, for 1 corresponding to $\text{ROOT}(p)$ and $|\text{MB}(p)|$ corresponding to $\text{OUT}(p)$. For a rank k, by $p(k)$ we denote any pattern isomorphic with the subtree of p rooted at the main branch node of rank k. By $node_p(k)$ we denote the node of rank k in the main branch of p.

A *prefix* p' of a tree pattern p is any tree pattern that can be built from p by setting $\text{ROOT}(p)$ as $\text{ROOT}(p')$, setting some node $n \in \text{MBN}(p)$ as $\text{OUT}(p')$, and removing all the main branch nodes descendants of n along with their predicates. A *suffix* p' of a tree pattern p is any subtree of p rooted at a node in $\text{MBN}(p)$.

We associate a name to each predicate in a pattern p (in lexicographic order). For a given predicate P, by n_P we denote the main branch node that is parent of P in q. By r_P we denote P's position on the main branch, i.e., the rank of the node n_P. By q_P we denote the pattern formed by the node n_P, as $\text{ROOT}(q_P)$, the pattern of P, and the edge connecting them. By $root_P$ we denote the node of p representing the root of P's pattern.

We also refer to the *tokens* of tree pattern p: more specifically, the main branch of p can be partitioned by its sub-sequences separated by //-edges, and each sub-pattern corresponding to such a sub-sequence is called a *token* of p. We can thus see a pattern p as a sequence of tokens (i.e., /-patterns) $p = t_1//t_2//\ldots//t_k$. We call t_1, the token starting with $\text{ROOT}(p)$, the *first token* of p. The token t_k, which ends by $\text{OUT}(p)$, is called the *last token* of p.

3. QUERY SET SPECIFICATIONS

We consider sets of XPath queries encoded using a grammar-like formalism, Query Set Specifications (QSS), similar to [14].

DEFINITION 3.1. *A* Query Set Specification *(QSS) is a tuple (F, Σ, P, S) where*

- *F is the set of tree fragment names*
- *Σ, with $\Sigma \cap F = \emptyset$ is the set of element names*
- *$S \in F$ is the start tree fragment name*
- *P is a collection of expansion rules of the form $f() \rightarrow tf$ or $f(X) \rightarrow tf$. where f is a tree fragment name, tf is a tree fragment and X denotes the output mark. Empty rules, of the form $f \rightarrow$ (no tree fragment) are also allowed.*

f is called the left-hand side (abbreviated as LHS) and tf is called the right-hand side (RHS) of the rule.

A tree fragment is a labeled tree that may consist of the following:

- *element nodes, labeled with symbols from Σ,*
- *tree fragment nodes n labeled with symbols from F,*
- *edges either of child type, denoted by simple lines, or of descendant type, denoted by double lines,*
- *the output mark X associated to one node (of either kind).*

In any rule, in the RHS one unique node may have the output mark (X) if and only if that rule has the output mark on the LHS.

As a notation convention, we serialize QSS tree fragments as *XP* expressions with an output mark (X), if present.

QSS expansions. A finite expansion (in short *expansion*) of a QSS $\mathcal{P}$ is any tree pattern p having a body obtained as follows:

- starting from a rule $S(X) \rightarrow tf$,
- apply on tf the following expansion step a finite number of times until no more tree fragment names are left: for some node n labeled by a tree fragment name f, pick a rule defining f (i.e., f is the LHS) and replace n by the RHS of that rule; if n has the output mark, use only rules with LHS $f(X)$.
- set the node having the output mark as $\text{OUT}(p)$.

We say that p is *generated* by $\mathcal{P}$. Note that the set of expansions can be infinite if the QSS is recursive.

DEFINITION 3.2 (EXPRESSIBILITY AND SUPPORT). *For an XP query q, a QSS $\mathcal{P}$, and a rewriting language $\mathcal{L}_R$ we say that*

1. *q is* expressible *by $\mathcal{P}$ iff q is equivalent to an expansion of $\mathcal{P}$.*
2. *q is* supported *by $\mathcal{P}$ in $\mathcal{L}_R$ iff there is a finite set $\mathcal{V}$ of XP queries generated by $\mathcal{P}$, with corresponding view documents $D_\mathcal{V}$, such that there is a rewriting of q formulated in $\mathcal{L}_R$ that navigates only in documents from $D_\mathcal{V}$.*

The definition of support given above depends on the language $\mathcal{L}_R$ in which the rewritings can be expressed. If rewritings are expressed in *XP*, then all one can do is navigate inside one view. However, if the source exposes persistent node ids, it becomes possible to intersect of view results. In this case, one can choose $\mathcal{L}_R$ to be $XP^\cap$ and use several views in more complex rewritings.

EXAMPLE 3.1. *The QSS $\mathcal{P}$ below generates queries returning information about museums that will be visited on a guided trip or as part of a tour in whose schedule there is also allotted time for taking a walk. Trips that appear nested are secondary trips.*

$$
\begin{array}{ll}
(\mathcal{P}) & f_0(X) \rightarrow \mathit{doc}(T)//\mathit{vacation}//f_1(X) \\
& f_1(X) \rightarrow \mathit{trip}/f_1(X) \\
& f_1(X) \rightarrow \mathit{trip}[\mathit{guide}]//\mathit{museum}(X) \\
& f_1(X) \rightarrow \mathit{trip}//\mathit{tour}[\mathit{schedule}//\mathit{walk}]/\mathit{museum}(X)
\end{array}
$$

It can be checked that v_1 and v_2 introduced before are among the expansions of $\mathcal{P}$. When considering v_1 and v_2 as user queries, we can also say they are expressed by $\mathcal{P}$.

Consider the following client query q_2, asking for museums that have temporary exhibitions and are visited in secondary trips:

$$q_2\text{: } \mathit{doc(T)//vacation//trip/trip[guide]//museum[temp]}.$$

q_2 is obviously not expressed by $\mathcal{P}$(there is no temp element node in $\mathcal{P}$). However, it is enough to filter the result of v_1 by the predicate [temp] to obtain the same result as q_2, hence q_2 is supported by $\mathcal{P}$:

$$q_2 \equiv \mathit{doc}(v_1)/v_1/\mathit{museum}[\mathit{temp}]$$

Consider the query q_1 of Example 1.1. One can check that q_1 cannot be answered by navigating into a single view. Suppose now that the views expose persistent node ids. By using Lemmas 2.1, 2.2, one can check that the support of q_1 is witnessed by v_1 and v_2:

$$q_1 \equiv \mathit{doc}(v_1)/v_1/\mathit{museum} \cap \mathit{doc}(v_2)/v_2/\mathit{museum}.$$

Intuitively, this holds because q_1 is one of the interleavings of v_1 and v_2 and all other interleavings are contained in q_1.

Normalization. For ease of presentation, we introduce first some normalization steps on the QSS syntax. First, the set of tree fragment names that have the output mark (denoted *unary*) is assumed disjoint from those that do not have it (denoted *boolean*). Second, we equivalently transform all rules such that, in any RHS, tree fragments have depth at most 1, and the nodes of depth 1 can only be labeled by tree fragment names (i.e., a RHS is a tree fragment formed by a root and possibly some tree fragment children, connected by either /-edges or //-edges to the root). For that, we may introduce additional tree fragment names. After normalization, for l being a label in Σ, $c_1, \ldots, c_n$, $d_1, \ldots, d_m$ being two (possibly empty) lists of tree fragment names and g being a tree fragment name as well, any non-empty rule falls into one of the following cases:

$$
\begin{array}{rcl}
f() & \rightarrow & l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()] \\
f(X) & \rightarrow & l(X)[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()] \\
f(X) & \rightarrow & l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()]/g(X) \\
f(X) & \rightarrow & l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()]//g(X)
\end{array}
$$

For any fragment name f and rule

$$f(X) \rightarrow l[c_1() \ldots c_n, .//d_1(), \ldots, .//d_m()]\ \mathit{edge}\ g(X),$$

by v_f we denote any possible expansion of f via that rule. By v'_f we denote any pattern that can be obtained from the rule by (i) expanding g into the empty pattern, and (ii) expanding the c_is and the g_js in some (any) possible way. Note that v'_f has only one main branch node (the root).

EXAMPLE 3.2. *The result of normalizing the QSS $\mathcal{P}$ from Example 3.1 is the following specification:*

$$
\begin{array}{ll}
f_0(X) \rightarrow \mathit{doc}(T)//f_1(X), & f_1(X) \rightarrow \mathit{vacation}//f_2(X) \\
f_2(X) \rightarrow \mathit{trip}/f_2(X), & f_2(X) \rightarrow \mathit{trip}[f_7()]//f_5(X) \\
f_2(X) \rightarrow \mathit{trip}//f_3(X), & f_3(X) \rightarrow \mathit{tour}[f_4()]/f_5(X) \\
f_4() \rightarrow \mathit{schedule}//f_6(), & f_5(X) \rightarrow \mathit{museum}(X) \\
f_6() \rightarrow \mathit{walk}, & f_7() \rightarrow \mathit{guide}
\end{array}
$$

4. EXPRESSIBILITY

We consider in this section the problem of expressibility: given a query q and a QSS $\mathcal{P}$ encoding a set of views, decide if there exists a view v generated by $\mathcal{P}$ that is equivalent to q.

Conceptually, in order to test expressibility, one has to enumerate the set of views and, for each view, check its equivalence to q. This is obviously unfeasible in our setting, since the set of views is potentially infinite. But the following observation delivers a naïve algorithm: only views that contain q have to be considered, and there are only finitely many distinct (w.r.t. isomorphism) candidates since containment mapping into q limits both the maximum length of a path (by the maximal path length in q) and the set of node labels (by the ones of q). Therefore, one can decide expressibility by enumerating all the candidate views and checking for each candidate if (a) it is equivalent to q, and (b) it is indeed an expansion of $\mathcal{P}$. However, this solution has limited practical interest beyond the fact that it shows decidability for our problem, since it is non-elementary in time complexity.

Our main contribution here is to provide a PTIME decision procedure for expressibility. The intuition behind our algorithm is the following. We do not enumerate expansions, and instead we group views and view fragments (which are assembled by the QSS to form a view) into *equivalence classes* w.r.t. their behavior in the algorithm for checking equivalence with q. Since there are fewer (only polynomially many) possible behaviors, manipulating such equivalence classes instead of explicit views or fragments thereof enables our PTIME solution.

As a compact representation for equivalence classes, we use *descriptors*. Informally, we use two kinds of descriptors for views or view fragments:

- *mapping descriptors*, which record if some expansion of a tree fragment name maps into a subtree of q,
- *equivalence descriptors*, which record if some expansion of a tree fragment name is equivalent to a subtree of q.

The rest of this section is organized as follows.

We first observe that equivalence for tree patterns is reducible to equivalence for a different flavor of patterns, *boolean tree patterns* ([11]). These are tree patterns of arity 0 (no output node) that test if evaluating a pattern over an XML document yields an empty result or not. Following this observation, for presentation simplicity, we solve expressibility for boolean tree patterns (Section 4.1).

Then, in Section 4.2, we show how expressibility for tree patterns (arity 1) can be reduced to expressibility for boolean tree patterns.

4.1 Expressibility for boolean tree patterns

We study in this section expressibility for boolean tree patterns. Their semantics, based on the same notion of embedding, can be easily adapted from the case of arity 1: the result of applying a boolean tree pattern p to an XML tree t is either the empty set $\emptyset$ or the set $\{\text{ROOT}(t)\}$. In the first case, we say that the result is *false*, in the latter, we say it is *true*. Containment and equivalence for boolean tree patterns are also based on mappings, with the only difference that there is no output node.

In the remainder of this section all patterns (queries and views) are boolean tree patterns. A QSS will have either rules of the form $f() \rightarrow l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()]$ or empty rules.

In order to clarify the role of descriptors and the equivalence classes they might stand for, let us first consider how one can test equivalence between a query q and view v. The classic approach for checking this is dynamic programming, bottom-up, using boolean matrices M that bookkeep mappings in both directions. $M(n_1, n_2)$ is *true* if the subtree rooted at n_1 contains the one rooted at n_2.

We prefer instead a variation on this approach, which will enable our PTIME solution. Since wildcard is not used, equivalence between a query q and a view v translates into q and v being *isomorphic* modulo minimization. Assuming that q is already minimized, this means that v has to be q plus some redundant branches, i.e.

- q is isomorphic to (part of) v, i.e. there is a containment mapping ψ from q into v, and the inverse ψ^{-1} is a partial mapping from v into q,
- the partial mapping ψ^{-1} can be completed to a containment mapping from v into q

In the above, no two nodes of q can have the same image under ψ. In other words, some nodes of v have an "equivalence role", and there must be one such node corresponding to each node of q, while the remaining nodes are redundant and it suffices to have only a "mapping role". This suggests that it is enough to build bottom-up only one matrix M, for containment from subtrees of v into subtrees of q, if in in parallel we bookkeep in another matrix details about *equivalence* between subtrees. A field in the equivalence matrix, $E(n_1, n_2)$, for $n_1 \in \text{NODES}(v)$, $n_2 \in \text{NODES}(q)$, indicates that the subtree $v(n_1)$ is equivalent with the subtree $q(n_2)$.

With these two matrices, checking $v \equiv q$ by a bottom-up pass is straightforward, by applying the following steps until fix-point:

Assuming that $M(n_1, n_2)$ and $E(n_1, n_2)$ are *true* for any leaf nodes $n_1 \in \text{NODES}(v)$, $n_2 \in \text{NODES}(q)$ having the same label,

A) For each pair (n_1, n_2), $n_1 \in \text{NODES}(v)$, $n_2 \in \text{NODES}(q)$ having the same label, set $M(n_1, n_2)$ to *true* if:

1. for each /-child n of n_1 there exists a /-child n' of n_2 s.t. $M(n, n') = true$,
2. for each //-child n of n_1 there exists a descendant n' of n_2 s.t. $M(n, n') = true$.

B) For each pair (n_1, n_2), $n_1 \in \text{NODES}(v)$, $n_2 \in \text{NODES}(q)$ having the same label, set $E(n_1, n_2)$ and $M(n_1, n_2)$ to *true* if:

1. for each /-child n of n_2 there exists a /-child n' of n_1 s.t. $E(n, n') = true$,
2. for each //-child n of n_2 there exists a descendant n' of n_1 s.t. $E(n, n') = true$,
3. for each /-child n of n_1 that was not referred to at step (1), there exists a /-child n' of n_2 s.t. $M(n, n') = true$,
4. for each //-child n of n_1 that was not referred to at step (2), there exists a descendant n' of n_2 s.t. $M(n, n') = true$.

We are now ready to present our PTIME algorithm for expressibility. We will adapt the above approach for testing equivalence, which builds incrementally (bottom-up, one level at a time) the mapping and equivalence details, to the setting when views are generated by a QSS by expanding fragment names. We will use *mapping* and *equivalence descriptors* to record for each tree fragment name if some of its expansions witnesses equivalence with or existence of mapping into a part of the query. More precisely,

DEFINITION 4.1. *For a fragment name f of a QSS $\mathcal{P}$, a* mapping descriptor *is a tuple map(f, n), where $n \in \text{NODES}(q)$, indicating that f has an expansion v_f in $\mathcal{P}$ that contains the subtree of q rooted at node n.*

An equivalence descriptor *is a tuple $equiv(f, n)$, where $n \in \text{NODES}(q)$, indicating that f has an expansion v_f in $\mathcal{P}$ that is equivalent with the subtree of q rooted at node n.*

Note that a descriptor *equiv*(f, n) will also tell us where the expansion it stands for maps (or not) in q. In other words, once we have an equivalence descriptor for a fragment name expansion, we can infer *all* mapping descriptors for it.

EXAMPLE 4.1. *Suppose that the data source publishes a modified version of the QSS from Example 3.2, enforcing the possibility of taking a walk on trips that contain tours. This translates into replacing the last rule for f_2 with the rule (unnormalized):*

$$f_2(X) \rightarrow \textit{trip}[.//f_6()]//f_3(X).$$

A client interface generates and sends a query identical to v_2 of Example 2.1 to this source.

The proof of expressibility will consist in finding an equivalence descriptor for the root of the tree pattern. To infer the existence of this descriptor, we compute descriptors going bottom up in the pattern and in the normalized QSS from Example 3.2.

We start with the leaves, for which we find $d_1 = equiv(f_5, n_{m2})$ and $d_2 = equiv(f_6, n_{w1})$, $d_2' = map(f_6, n_{w1})$. Using d_2, we can infer the descriptor $d_3 = equiv(f_4, n_{s1})$, which, together with the descriptor for n_{m2}, enables a descriptor $d_4 = equiv(f_3, n_{to1})$. Since n_{w1} is a descendant of n_{tr3}, we can use the mapping descriptor d_2' and the equivalence descriptor d_4 to build a descriptor $equiv(f_2, n_{tr3})$. This in turn enables a descriptor $equiv(f_1, n_{v2})$, which leads to inferring a descriptor for the root: $equiv(f_0, n_{d2})$.

Thus we can check that expressibility holds, even if v_2 is not isomorphic to any expansion of the QSS (since it has no predicate on the node labeled with trip).

Our algorithm for testing expressibility will mimic the two steps (A) and (B) above, applying them instead on QSS rules and fragment nodes via descriptors. Given descriptors for the fragment names in the RHS, we will infer new descriptors for the fragment name on the LHS. The only notable difference with respect to the approach for checking equivalence is for steps (B.1) and (B.2). For a fragment name f and node $n \in \text{NODES}(q)$, fragment names children of f in a rule may have several $equiv$ descriptors, referring to different nodes of q. We must choose one among them in a way that

does not preclude the inference of a descriptor $equiv(f, n)$, when one exists. For that, we will use a function tf-cover, which takes as input a set of nodes N, a set of tree fragment names C and an array L such that for every $n \in N$, $L(n) \subseteq C$. It returns *true* if there is a way to pick a distinct tree fragment name from each $L(n)$, for all $n \in N$. This function is based on a max-flow computation and its running time is $O((|C| + |N|) * |C|)$. We refer the reader to the extended version of this paper [7], for the detailed definition of tf-cover.

The computation of descriptors (algorithm findDescExpr) starts with productions without tree fragment nodes on the RHS and continues inferring descriptors until a fixed point is reached. It runs in polynomial time because (a) there are only polynomially many descriptors (their number is proportional to the size of the QSS multiplied by the size of the query), and (b) each incremental, bottom-up step for inferring a new descriptor runs in polynomial time.

Algorithm findDescExpr(q, $\mathcal{P}$)**:**

A. Start with an empty set of descriptors R.

B. For each rule $f() \rightarrow ()$, node $n \in \text{NODES}(q)$, add to R the descriptor *map*(f, n).

C. For each rule $f() \rightarrow l$ (i.e., the RHS has only one node) and each node $n \in \text{NODES}(q)$ labeled by l, add to R the descriptors *equiv*(f, n) and *map*(f, n).

Repeat until R unchanged:

D. For each rule $f() \rightarrow l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()]$, add to R a descriptor *map*(f, n) if n is labeled by l and

- for each fragment name c_i there exists a descriptor *map*(c_i, n') s.t. n' is a /-child of n,
- for each fragment name d_j there exists a descriptor *map*(d_j, n') s.t. n' is a descendant of n.

E. for each rule $f() \rightarrow l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()]$: add to R the descriptors *equiv*(f, n) and *map*(f, n) if

1. tf-cover(N_1, C, L) returns *true*, where N_1 is the set of /-children of n, $C \subseteq \{c_1, \ldots, c_n\}$ is the set of fragment names that have a descriptor *equiv*(c_i, n') for $n' \in N_1$ and, for each $n' \in N_1$, $L(n') \subseteq C$ is the set of fragments names that have a descriptor *equiv*(c_i, n').
2. tf-cover(N_2, D, L) returns *true*, where N_2 is the set of //-children of n, $D \subseteq \{d_1, \ldots, d_m\}$ is the set of fragment names that have a descriptor *equiv*(d_j, n') for $n' \in N_2$ and, for each $n' \in N_2$, $L(n') \subseteq D$ is the set of fragments names that have a descriptor *equiv*(d_j, n').
3. for each fragment name $c_i \notin C$, there exists a descriptor *map*(c_i, n') s.t. n' is a /-child of n,
4. for each fragment name $d_j \notin D$ there exists a descriptor *map*(d_j, n') s.t. n' is a descendant of n.

THEOREM 4.1. *A boolean tree pattern q is expressed by a QSS $\mathcal{P}$ iff* findDescExpr*$(q, \mathcal{P})$ outputs a descriptor $equiv(S, \text{ROOT}(q))$, for S being the start fragment name of $\mathcal{P}$.* findDescExpr *runs in polynomial time in the size of the query and of the QSS.*

Remark. The assumption that the input query q is minimized - which implies that no two nodes of q can have the same image under the ψ function described above - is important for our algorithm. It allows us to avoid a bottom-up approach that might also have to bookkeep mappings from the query into the views. This would require descriptors that pair *a set of subtrees* of q with an expansion, leading to a worst-case exponentially large space for descriptors.

4.2 Expressibility for tree patterns

We now consider expressibility for standard tree pattern queries (patterns with an output node).

It is well known from previous literature that problems such as tree pattern containment and equivalence reduce to containment, respectively equivalence, for boolean patterns. This is based on the following translation: let s be a new label (from *selection*), for a tree pattern p let p_0 denote the boolean tree pattern obtained from p by (i) adding a /-child labeled s below the output node of p, and (ii) removing the output mark. From [11], for two tree patterns p and p', we have that $p \equiv p'$ iff $p_0 \equiv p'_0$.

A similar transformation can be applied for expressibility. Given a QSS $\mathcal{P}$, let $\mathcal{P}_0$ be the QSS obtained from $\mathcal{P}$ by (i) plugging a /-child labeled s below each node having an explicit label and the output mark, and (ii) making all rules and tree fragment names boolean by removing their output mark. $\mathcal{P}_0$ generates boolean tree patterns and, since $\mathcal{P}$'s sets of unary and boolean tree fragment names were assumed disjoint, $\mathcal{P}_0$'s expansions have exactly one s-labeled node. We can prove the following:

THEOREM 4.2. *A tree pattern query q is expressed by a QSS $\mathcal{P}$ iff the boolean tree pattern q_0 is expressed by the QSS $\mathcal{P}_0$.*

5. SUPPORT

For the problem of support, the fact whether the source enables persistent node ids (that are then exposed in query results) or not has a significant impact on the rewrite plans one can build. In both settings, with or without node ids, rewriting under an explicitly listed set of views has been studied in previous literature. We will now revisit them for support.

In the first setting, the identity of the nodes forming the result of a query is not exposed in results. By consequence, the only possible rewrite plans consist in accessing a view result and maybe navigating inside it (via query *compensation*). This setting was considered in [17], and the rewriting problem was shown to be in PTIME for *XP*. We study support in the absence of ids in Section 5.1. Our main result here is that support reduces to expressibility, which allows us to reuse the PTIME algorithm given in Section 4.

In the second setting, for which rewriting under an explicit set of views was studied in [5], data sources expose persistent node ids. This enables more complex rewrite plans, in which the *intersection* of view results plays a crucial role. We revisit this setting, for the support problem, in Section 6. As our general approach, we will apply the same kind of reasoning that was used for expressibility. We will group views into equivalence classes w.r.t. crucial tests for support and we will manipulate classes (encoded as *view descriptors*) instead of explicit views. This will enable us to avoid the enumeration of a potentially large space of views and rewrite plans.

5.1 Support in the absence of ids

When persistent identifiers are not exposed, a rewrite plan consist in accessing a view's result and maybe navigating inside it, and this navigation is called *compensation*. This is why expressibility and support in the absence of ids remain strongly related, as support simply amounts to finding a candidate view v which, via compensation, becomes equivalent with the input query.

Let us first introduce as notation for this operation the compensate function, which performs the concatenation operation from [17], by copying extra navigation from the query into the rewrite plan. For a view $v \in XP$, an input query q, and a main branch rank k in q, $\text{compensate}(v, q, k)$ returns the query obtained by deleting the first symbol from $x = xpath(q(k))$ and concatenating the rest to v. For instance, the result of compensating v = a/b with x = b[c][d]/e is the concatenation of a/b and [c][d]/e, i.e. a/b[c][d]/e.

We can reformulate the result from previous literature as follows:

THEOREM 5.1 ([17]). *Given a set of explicit views $\mathcal{V}$, a query q can be answered by $\mathcal{V}$ if and only if there exists a view v and main branch rank k in q such that* $\text{compensate}(v, q, k) \equiv q$.

Going now to views encoded as QSS expansions, we reduce the problem of support to expressibility, following the idea that support amounts to expressibility by a certain "compensated" specification.

From a given QSS $\mathcal{P}$, we will build a new QSS that generates, besides $\mathcal{P}$'s expansions, all their possible compensated versions w.r.t. q. More precisely, given an input query q and a QSS $\mathcal{P}$, let $comp(\mathcal{P}, q)$ denote the QSS obtained from $\mathcal{P}$ as follows:

For any rule yielding the output node, i.e., of the form

$$f(X) \rightarrow l(X)[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()],$$

for each rank k in q, add a new rule, of the form (with a little departure from the normalized QSS syntax):

$$f(X) \rightarrow \text{compensate}(l[c_1(), \ldots, c_n(), .//d_1(), \ldots, .//d_m()], q, k)$$

We can prove the following:

THEOREM 5.2. *A query q is supported by a QSS $\mathcal{P}$ if and only if it is expressed by the QSS $comp(\mathcal{P}, q)$.*

EXAMPLE 5.1. *An example of support in the absence of persistent ids has already been given in Example 3.1: q_2 can be rewritten by compensating v_1 with a temp predicate.*

6. SUPPORT IN THE PRESENCE OF IDS

We consider in this section the problems of support in the presence of node ids, denoted in the following *id-based support*. First, deciding the existence of a rewriting for an *XP* query under an explicit set of *XP* views becomes coNP-hard, as it was shown in [5].

THEOREM 6.1 ([5]). *Testing if an XP query can be rewritten using explicitly listed views, in the presence of ids, is coNP-hard.*

As a corollary, it follows immediately that the same lower-bound holds for id-based support.

COROLLARY 6.1. *Id-based support for XP is coNP-hard.*

Since our focus is on efficient algorithms for support, we next investigate the tightest restrictions for tractability. We consider the fragment of *extended skeletons* (XP_{es}), for which the rewriting problem was shown tractable in [5]. The restrictions imposed by the XP_{es} fragment on the input query were shown to be necessary for tractability, as their relaxation leads to coNP-hardness. It is therefore natural to ask whether the support problem is also tractable for input queries from this fragment. Note that one cannot do better, i.e., obtain a decision procedure for queries outside this fragment, since the problem of support subsumes the rewriting problem.

The remainder of this paper is thus dedicated to studying support for extended skeletons, focusing on efficient (PTIME) solutions that are sound in general (i.e., for any *XP* input query) and complete under fairly general conditions, and this *without restricting the language of views* (which remains *XP*). We show that id-based support exhibits a complexity dichotomy: the sub-fragment of XP_{es} representing queries that have at least one //-edge in the main branch, denoted hereafter *multi-token*, continues to be in PTIME (Theorem 7.5), but the complementary sub-fragment that represents queries with only /-edges in the main branch, denoted hereafter *single-token*, interestingly, is NP-hard (see Theorem 8.1).

The fragment of multi-token queries is particularly useful in practice since often, for reasons such as conciseness or generality in the presence of schema heterogeneity, one does not want to write in a query all the navigation steps over a document (may skip some steps by //). After defining the fragment of extended skeletons, we consider in Section 7 support for multi-token queries and, in Section 8, support for single-token queries.

Extended skeletons (XP_{es}). Intuitively, this fragment limits the use of //-edges in predicates, in a way which can be summarized as follows: a token t of a pattern p will not have predicates that may become redundant because of descendants of t and their respective predicates in some interleaving p might be involved in.

Let us first introduce some additional terminology. By a *//-subpredicate st* we denote a predicate subtree whose root is connected by a //-edge to a linear /-path l that comes from the main branch node n to which st is associated (as in $n[\ldots[.//st]]$). l is called the *incoming /-path* of st and can be empty.

Extended skeletons are patterns having the following property: for any main branch node n and //-subpredicate st of n, there is no mapping (in either direction) between the code of the incoming /-path of st and the one of the /-path following n in the main branch (where the empty code is assumed to map in any code).

For instance, expressions such as a[b//c]/d//e or a[b//c//d]/e//d are in XP_{es}, while a[b//c]/b//d, a[b//c]//d, a[.//b]/c//d or a[.//b]//c are not. XP_{es} does not restrict in any way the usage of //-edges in the main branch or the usage of predicates with /-edges only.

7. MULTI-TOKEN QUERIES

We consider now id-based support for XP_{es} multi-token queries. For presentation simplicity, we first limit the discussion to rewrite plans that are intersections of views (no compensation before the intersection step). General $XP^{\cap}$ plans, i.e., intersections of possibly compensated views, are considered in Section 7.4.

As in the case of expressibility, we think of views as grouped into equivalence classes w.r.t. to crucial tests for support. We manipulate such classes, represented by *view descriptors*, instead of explicit views, avoiding the enumeration of a potentially large space of views and plans. As a QSS constructs views by putting together fragments, we construct view descriptors from *fragment descriptors*, which represent equivalence classes for fragment expansions.

This section is organized as follows. In order to clarify the role of view descriptors and the equivalence classes they stand for, we first revisit in Section 7.1 the PTIME algorithm of [5] for deciding if an XP_{es} multi-token query q can be rewritten by an intersection of explicit *XP* views $\mathcal{V}$ already known to contain q. That algorithm was based on applying DAG-pattern rewrite steps towards a tree pattern and then checking equivalence with q. We reformulate it into an algorithm (testEquiv) that applies individual tests on the view definitions instead. Then, in Section 7.2, we introduce equivalence classes for views w.r.t. the tests of testEquiv, and *view descriptors* as a means to represent such classes. We reformulate the testEquiv algorithm into a new algorithm, testEquivDesc, that runs on view descriptors instead of explicit view definitions. Finally, in Section 7.3 we give a PTIME sound and complete algorithm for computing descriptors for the expansions of a QSS.

7.1 Rewriting with an explicit set of views

Let the input XP_{es} multi-token query q be of the form $q = ft//m//lt$, where ft denotes the first token, lt denotes the last token and m denotes the rest (m may be empty).

Let $\mathcal{V} = \{v_1, \ldots, v_n\}$ denote a set of *XP* views such that $q \sqsubseteq v_i$ for each v_i. Let each view v_i be of the form $v_i = ft_i//m_i//lt_i$.

For an *XP* query v, by its *extended skeleton*, we denote the XP_{es} query obtained by pruning out all the //-subpredicates violating the XP_{es} condition. We can prove the following auxiliary lemma:

LEMMA 7.1. *An XP_{es} query is equivalent to an intersection of views iff equivalent to the intersection of their extended skeletons.*

By Lemma 7.1, w.l.o.g. all views are assumed hereafter from XP_{es}.

Notation. Let $ft_{\mathcal{V}}$ denote the query obtained by "combining" the first tokens $ft_1, \ldots, ft_n$ as follows: start by coalescing the roots, then continue coalescing top-down any pair of main branch nodes that have the same parent and label. This process yields a tree because each first token ft_i maps in the first token of q, ft, hence each $\text{MB}(ft_i)$ is a prefix of $\text{MB}(ft)$. Let $lt_{\mathcal{V}}$ denote the query obtained by "combining" $lt_1, \ldots, lt_n$ similarly: start by coalescing the output nodes, then continue by coalescing bottom-up any pair of main branch nodes that have a common child and the same label.

EXAMPLE 7.1. *For instance, for two views* $\mathcal{V} = \{v', v''\}$,

$$v' = doc(T)/vacation/trip[guide]//tour/museum,$$
$$v'' = doc(T)/vacation[.//walk]//museum[gallery],$$

the result of combining their first tokens, respectively last tokens is

$$ft_{\mathcal{V}} = doc(T)/vacation[.//walk]/trip[guide],$$
$$lt_{\mathcal{V}} = tour/museum[gallery].$$

Given $\text{MB}(ft)$, $\text{MB}(lt)$, if there exists a minimal (non-empty) prefix of $\text{MB}(lt)$ that is isomorphic with a suffix of $\text{MB}(ft)$, let $\text{MB}(lt)'$ denote the pattern obtained from $\text{MB}(lt)$ by cutting out this prefix. Then, let l_q denote the linear pattern $\text{MB}(ft)/\text{MB}(lt)'$. If l_q is undefined by the above, by convention it is the empty pattern.

EXAMPLE 7.2. *For instance, for the query*

$$q = doc(T)/vacation[.//walk]/tour//tour/museum,$$

l_q *is well-defined, as* $l_q = doc(T)/vacation/tour/museum$.

Given $\text{MB}(ft)$ and $\text{MB}(m)$, if there exists a minimal (non-empty) suffix of $\text{MB}(ft)$ that is isomorphic with a prefix of $\text{MB}(m)$, let $\text{MB}(ft)_m$ denote the pattern obtained from $\text{MB}(ft)$ by cutting out this suffix. If $\text{MB}(ft)_m$ is undefined by the above, by convention it is the empty pattern. Similarly, given $\text{MB}(lt)$ and $\text{MB}(m)$, if there exists a minimal (non-empty) prefix of $\text{MB}(lt)$ that is isomorphic with a suffix of $\text{MB}(m)$, let $\text{MB}(lt)_m$ denote the pattern obtained from $\text{MB}(lt)$ by cutting out this prefix. If $\text{MB}(lt)_m$ is undefined by the above, by convention it is the empty pattern.

We are now ready to present our reformulation of the PTIME algorithm of [5], which will test that $\cap\mathcal{V} \sqsubseteq q$. By Lemma 2.2, q must contain each possible interleaving i of the set $\mathcal{V}$ or, in other words, for each $i \in interleave(\mathcal{V})$ the following should hold:

- the first token of q can be mapped in the first token of i s.t. the image of $\text{ROOT}(q)$ is $\text{ROOT}(i)$,
- the last token of q can be mapped in the last token of i s.t. the image of $\text{OUT}(q)$ is $\text{OUT}(i)$,
- the images of these two tokens in i are disjoint,
- the intermediary part m (if non-empty) of q can be mapped somewhere between these two images in i.

Algorithm 1 testEquiv($\mathcal{V}, q$)

1: **let each** $v_i = ft_i//m_i//lt_i$, **let** $q = ft//m//lt$
2: **compute** the patterns $ft_{\mathcal{V}}$, $lt_{\mathcal{V}}$, l_q, $\text{MB}(ft)_m$ and $\text{MB}(lt)_m$
3: **if** $ft_{\mathcal{V}} \equiv ft$ and $lt_{\mathcal{V}} \equiv lt$ **then**
4: **if** m is empty **then for each** $v_i \in \mathcal{V}$
5: **if** $\text{MB}(v_i)$ does not map into l_q **then** output `true`
6: **else** (m non-empty) **for each** $v_j \in \mathcal{V}$
7: **if** v_j can be seen as $prefix_j//m'//suffix_j$ **s.t.**
8: $m' \equiv m$
9: $prefix_j$ root-maps into ft, $suffix_j$ output-maps into lt
10: $\text{MB}(prefix_j)$ does not root-map into $\text{MB}(ft)_m$
11: $\text{MB}(suffix_j)$ does not output-map into $\text{MB}(lt)_m$
12: **then** output `true`

THEOREM 7.1. *For a multi-token XP query* q *and a set of XP views* $\mathcal{V}$, testEquiv *is a sound PTIME procedure for testing* $q \equiv \cap\mathcal{V}$.

For input queries from XP_{es} we can also prove completeness:

THEOREM 7.2. *For an* XP_{es} *multi-token query* q *and a set of XP views* $\mathcal{V}$, testEquiv *is complete for testing* $q \equiv \cap\mathcal{V}$.

7.2 View descriptors

We detail now how one can perform the tests of algorithm testEquiv even when abstracting away from the view definitions. The key idea is that one does not need the complete definitions but only the details used in these tests. With respect to these details, views can be seen as grouped into equivalence classes and views from the same class will be equally useful in the execution of the algorithm. This idea will be exploited by our *view descriptors*. We then reformulate testEquiv in terms of view descriptors in algorithm testEquivDesc. More precisely, assuming we are dealing with expansions of a QSS $\mathcal{P}$ with start fragment name S,

For line 3 of testEquiv**.** For the part $ft_{\mathcal{V}} \equiv ft$: a *first-token descriptor* will be a tuple $\mathbf{ft}(\mathbf{S}, \mathbf{p})$, where p denotes any pattern that can be built from a prefix of q's first token ft by removing all its predicates, except eventually for one. Such a descriptor indicates that there exists an expansion v s.t. $q \sqsubseteq v$ and v's first token is of the form p, plus eventually other predicates (ignored in the descriptor). These descriptors represent partitions (equivalence classes) of the space of views containing q w.r.t. their first tokens and the predicates on them. Each view belongs to at least one such class, but may belong to several of them (for different choices of predicates).

For the part $lt_{\mathcal{V}} \equiv lt$: a *last-token descriptor* is a tuple $\mathbf{lt}(\mathbf{S}, \mathbf{p})$, where p denotes any pattern that can be built from a suffix of q's last token lt by removing all its predicates, except eventually for one. Such a descriptor says that there is an expansion v s.t. $q \sqsubseteq v$ and v's last token is of the form p, plus eventually other predicates.

It is easy to see that the $\mathbf{ft}$ and $\mathbf{lt}$ view descriptors allow us to compute the patterns $ft_{\mathcal{V}}$ and $lt_{\mathcal{V}}$, provided they verify $ft_{\mathcal{V}} \equiv ft$ and $lt_{\mathcal{V}} \equiv lt$, without requiring the actual first and last tokens. The domain of these descriptors is quadratic in the size of q.

For line 5 of testEquiv**.** An *l-descriptor* is a tuple $\mathbf{l}(\mathbf{S})$, indicating that there exists an expansion v verifying $q \sqsubseteq v$ and $l_q \not\sqsubseteq \text{MB}(v)$. (This type of descriptor is an alias for the condition of line 5, denoting a partition of the space of views in two complementary classes.)

For lines 7-11 of testEquiv**.** An *m-descriptor* is a tuple $\mathbf{m}(\mathbf{S})$, indicating that there exists an expansion v verifying $q \sqsubseteq v$ and all the conditions of lines 7-11.

We now reformulate testEquiv into an algorithm that runs on a set of view descriptors $\mathcal{D}$, instead of the explicit views $\mathcal{V}$ to which they correspond. Unsurprisingly, the new algorithm follows closely the steps of testEquiv, since descriptors are tailored to its various tests.

Algorithm 2 testEquivDesc($\mathcal{D}, q$)

1: **from all** descriptors $\mathbf{ft}(\mathbf{S}, \mathbf{p}) \in \mathcal{D}$ **compute** the pattern $ft_{\mathcal{V}}$
2: **from all** descriptors $\mathbf{lt}(\mathbf{S}, \mathbf{p}) \in \mathcal{D}$ **compute** the pattern $lt_{\mathcal{V}}$
3: **if** $lt_{\mathcal{V}} \equiv ft$ and $lt_{\mathcal{V}} \equiv ft$ **then**
4: **if** m is empty **then**
5: **if** there exists a descriptor $\mathbf{l}(\mathbf{S}) \in \mathcal{D}$ **then** output `true`
6: **else if** there exists a descriptor $\mathbf{m}(\mathbf{S}) \in \mathcal{D}$ **then** output `true`

THEOREM 7.3. *For an XP query* q, *a finite set of XP views* $\mathcal{V}$ *and their corresponding descriptors* $\mathcal{D}$, testEquiv$(q,\mathcal{V})$ *outputs* true *if and only if* testEquivDesc$(q, \mathcal{D})$ *does so.*

EXAMPLE 7.3. *For the query* q_1 *in Example 1.1,* $ft = doc(T)$, $m = vacation//trip/trip[guide]$, $lt = tour[schedule//walk]/museum$.

For the QSS $\mathcal{P}$ *from Example 3.1 and its two expansions* v_1 *and* v_2, v_1 *can be represented by the descriptors* $\mathbf{ft}(S, doc(T))$, $\mathbf{lt}(S, museum)$, $\mathbf{m}(S)$ *too since* v_1 *has the form* $pref_1//m//suff_1$, *with* $pref_1 = doc(T)$ *and* $suff_1 = museum$. *Similarly,* v_2 *is represented by* $\mathbf{ft}(S, doc(T))$ *and* $\mathbf{lt}(S, tour[schedule//walk]/museum)$.

Running on these descriptors, testEquivDesc *will confirm that there exists an equivalent rewriting for* q_1 *using* $\{v_1, v_2\}$.

7.3 View descriptors from a QSS

We present in this section a bottom-up algorithm (findDescSupp) that runs on a QSS $\mathcal{P}$ and a multi-token query q, computing the view descriptors (w.r.t. q) for the expansions of $\mathcal{P}$. Our algorithm is sound and complete, running in polynomial time. Via Theorems 7.3 and 7.1, findDescSupp delivers a sound PTIME algorithm for support when the input queries are multi-token from *XP*. Moreover, via Theorems 7.3 and 7.2, it delivers a PTIME decision procedure for support when the input queries are multi-token from XP_{es}.

We will describe findDescSupp by separate subroutines, one for each of the four kinds of view descriptors (first-token descriptors in Section 7.3.1, last-token descriptors in Section 7.3.2, l-descriptors in Section 7.3.3 and m-descriptors in Section 7.3.4).

Since a QSS constructs views by putting together fragments, we construct our view descriptors via *fragment descriptors*, which represent equivalence classes for fragment expansions. Intuitively, fragment descriptors bookkeep in the bottom-up procedure certain partial details, on the expansions of fragment names, details that allow us to test *incrementally* the various conditions of testEquiv.

To better clarify our choices for fragment descriptors, let us first detail how the tests of testEquiv can be done in incremental manner.

Mapping and equivalence tests are naturally done bottom-up, one node at time, and this translates easily into procedures that run on the QSS and rely on fragment descriptors. We already presented in Section 4 how one can test in this way the existence of containment or equivalence with q or parts thereof. We will handle the tests of lines 3, 8 and 9 in testEquiv similarly, by descriptors which record mapping or equivalence details.

For line 5, the non-existence of a containment mapping between linear paths needs a slightly different approach. One can test incrementally if a linear path l_1 is contained in a linear path l_2 as follows:

- test if the last token of l_2 maps in the last token of l_1, such that $\text{OUT}(l_1)$ is the image of $\text{OUT}(l_2)$. Let k denote the start rank (the upmost node) of this mapping image.
- bottom-up, for each intermediary token t of l_2, map t in the *lowest possible*[1] available (i.e. above k) part of l_1. If no such mapping exists, we can conclude the non-existence of a containment mapping from l_2 in l_1. At each step, bookkeep as k the start rank of that image of t in l_1.
- finally, if the previous set of steps did not yield a negative answer already, a containment mapping of l_2 in l_1 does not exist if and only if the first token of l_2 cannot be mapped in l_1 s.t. (i) $\text{ROOT}(l_1)$ is the image of $\text{ROOT}(l_2)$, and (ii) the image of this first token of l_2 is above the current rank k.

A similar incremental approach, advancing one token at a time, can be used for the tests in lines 10 and 11, as we are dealing again with linear patterns. More precisely, a bottom-up approach as above can be used in the case of $\text{MB}(\textit{suffix}_j)$ and, symmetrically, a top-down one can be used in the case of $\text{MB}(\textit{prefix}_j)$.

Note that the approach above advances one token at a time, and not one node at a time (which would have fitted nicely with how views are built in a QSS). This is because we need to check that all possible partial mappings fail sooner or later to go through to a full containment mapping (for line 5), root-mapping (for line 10), respectively output-mapping (for line 11). And the only way to ensure that no mapping opportunity is prematurely discarded is to settle on a mapping image in a descriptor, the lowest possible one, only when a token is complete (i.e., its incoming edge is //).

We are now ready to detail how view descriptors are computed in the algorithm findDescSupp. We start by assuming that all *equiv* or *map* descriptors are pre-computed for the boolean fragment names (as described in Section 4). In the same style, we compute *containment* and *equivalence descriptors* for *unary fragment names* (i.e. those with an output mark). More precisely, a descriptor $contain(f, n)$, for $n \in \text{MBN}(q)$, (resp. $equiv(f, n)$) denotes that some expansion v_f contains (resp. is equivalent to) the suffix of q rooted at the main branch node n. Other types of fragment descriptors will be introduced next. For space reasons, examples illustrating the step-by-step computation of descriptors are given in [7].

[1]As we handle one token at a time, choosing the lowest available mapping image preserves all opportunities to find containment.

7.3.1 Computing first-token descriptors

For this part, we will use *prefix descriptors* for fragment names:

DEFINITION 7.1. Syntax: *For a unary fragment name f, a* prefix descriptor *is a tuple pref(f, p, k), for k being a rank in the range 1 to $|\text{MB}(ft)|$ and p denoting any pattern that can be obtained from ft by keeping (a) a substring of the main branch, starting from rank k, and (b) eventually, one predicate on that substring.*

Semantics: *There exists an expansion v_f s.t. (a) v_f has a containment mapping in the subtree of q rooted at the ft node of rank k, and (b) v_f has a first token which is of the form p plus additional predicates, if any (they are ignored in the descriptor).*

Step 1 **of** findDescSupp(q, $\mathcal{P}$)**.** Iterate the following steps:

1. For $f(X) \rightarrow l[c_1(), \ldots c_n(), .//d_1(), \ldots, .//d_m()]//g(X)$, add a prefix descriptor $\mathbf{pref}(\mathbf{f}, \mathbf{l}, \mathbf{k})$ for each rank k, $1 \leq k \leq |\text{MB}(ft)|$, s.t. $node_q(k)$ has label l, for which we can infer that v_f contains the pattern $q(k)$, by the following tests:
   - for each fragment name c_i there exists a descriptor $map(c_i, n)$, for n being a /-child of $node_q(k)$,
   - for each fragment name d_j there exists a descriptor $map(d_j, n)$, for n being a descendant of $node_q(k)$
   - there exists a containment descriptor $contain(g, n)$ for n being any main branch node of rank $k' > k$ in q.

   Moreover, if for a /-predicate (resp. //-predicate) P on $node_q(k)$ we have a descriptor $equiv(c_i, root_P)$ (resp. $equiv(d_j, root_P)$), add the descriptor $\mathbf{pref}(\mathbf{f}, \mathbf{l}[\mathbf{P}], \mathbf{k})$.
2. For $f(X) \rightarrow l[c_1(), \ldots c_n(), .//d_1(), \ldots, .//d_m()]/g(X)$, given a prefix descriptor $pref(g, p', k')$, add a prefix descriptor $\mathbf{pref}(\mathbf{f}, \mathbf{l}/\mathbf{p'}, \mathbf{k})$, for $k = k' - 1$, if $node_q(k)$ has label l and we can infer that v_f contains $q(k)$, as follows:
   - for each fragment name c_i there exists a descriptor $map(c_i, n)$, for n being a /-child of $node_q(k)$,
   - for each fragment name d_j there exists a descriptor $map(d_j, n)$, for n being a descendant of $node_q(k)$

   Moreover, if for a /-predicate (resp. //-predicate) P on $node_q(k)$ we have a descriptor $equiv(c_i, root_P)$ (resp. $equiv(d_j, root_P)$), add also $\mathbf{pref}(\mathbf{f}, \mathbf{l}[\mathbf{P}]/\text{MB}(p'), \mathbf{k})$.
3. Whenever a descriptor *pref(f,p,1)* is obtained, for $f = S$, add $\mathbf{ft}(\mathbf{S}, \mathbf{p})$ to the set of view descriptors.

7.3.2 Computing last-token descriptors

We use for this part two kinds of fragment descriptors: *suffix descriptors* and *full-suffix descriptors*.

DEFINITION 7.2. Syntax: *For a unary fragment name f, a* suffix descriptor *is a tuple suff(f, p), for p denoting any pattern that can be obtained from lt by keeping (a) a suffix of its main branch, and (b) eventually, one predicate on that suffix.*

Semantics: *This descriptor says that (a) v_f is a single-token query, of the form p plus maybe other predicates (ignored by the descriptor), and (b) v_f contains the subtree of lt rooted at the main branch node of rank $|\text{MB}(lt)| - |\text{MB}(p)| + 1$.*

DEFINITION 7.3. Syntax: *For a unary fragment name f, a* full-suffix descriptor *is a tuple fsuff(f, p, k), for k denoting a rank in q, and p being a pattern as defined in Definition 7.2 above.*

Semantics: *There exists an expansion v_f s.t. (a) v_f has a last token of the form p plus other predicates (if any), and (b) v_f maps in the subtree of q rooted at the main branch node of rank k.*

Step 2 **of** findDescSupp(q, $\mathcal{P}$)**:**

We compute *suffix descriptors* similarly to the prefix ones. From them, *full-suffix descriptors* are then computed bottom-up, by simple containment mapping checks. If a descriptor *fsuff(f, p, 1)* is obtained, for $f = S$, we add $\mathbf{lt}(\mathbf{S}, \mathbf{p})$ to the set of view descriptors. (For the explicit steps we refer the reader to [7].)

7.3.3 Computing l-descriptors

We have seen in Section 7.3 an incremental procedure that tests the non-existence of a containment mapping for linear patterns bottom-up, one token at a time. To run a similar test directly on the QSS (whose expansions are revealed one node at a time), we need additional bookkeeping, allowing us to chose mapping images one token at a time. For this, we record at each step in the bottom-up process the following: (i) the current first token of v_f, (ii) the *lowest possible* mapping image for the rest of v_f (except its first token). This allows us to settle on the lowest possible mapping (in a descriptor) only when the token is complete (we have its incoming edge and it is a //-edge). To this end, we use *partial l-descriptors*.

DEFINITION 7.4. Syntax: *For a unary fragment name f, a* partial l-descriptor *is a tuple $pl[f, k_1, (k_2, p)]$, where k_1 is a rank in q, k_2 is a rank in l_q and p is any substring of l_q.*

Semantics: *There exists an expansion v_f s.t. (a) v_f contains the subtree of q rooted at the main branch node of rank k_1, (b) the main branch of the first token of v_f is p, and (c) k_2 is the start (upmost rank) of the* lowest possible output-mapping image *of the rest of the main branch of v_f (i.e., except the first token, represented by p) into l_q. By convention, this rank is $|l_q| + 1$ when v_f has only one token (the one described by p) and is 0 when there is no such mapping.*

Step 3 **of** findDescSupp(*q*, $\mathcal{P}$)**.** Iterate the following steps:

1. For rules $f(X) \rightarrow l(X)[c_1(), \ldots c_n(), .//d_1(), \ldots, .//d_m()]$, if we can infer that v_f contains the subtree of q rooted at $\text{OUT}(q)$, add a descriptor $\mathbf{pl}[\mathbf{f}, |\text{MB}(q)|, (|\mathbf{l_q}| + \mathbf{1}, \mathbf{l})]$
2. For $f(X) \rightarrow l[\ldots]/g(X)$, given a descriptor $pl[g, k_1', (k_2', p')]$, if we can infer that v_f contains the pattern $q(k_1' - 1)$:
   - if f is not the start fragment name, add the descriptor $\mathbf{pl}[\mathbf{f}, \mathbf{k_1'} - \mathbf{1}, (\mathbf{k_2'}, \mathbf{l/p'})]$.
   - otherwise, if there is no mapping of l/p' into l_q whose image starts at $\text{ROOT}(l_q)$ and ends above k_2', add the descriptor $\mathbf{l}(\mathbf{S})$ to the set of view descriptors.
3. For $f(X) \rightarrow l[\ldots]//g(X)$, descriptor $pl[g, k_1', (k_2', p')]$, for each rank k_1, $1 \leq k_1 < k_1'$, s.t. we can infer that
   - if f is not the start fragment name, find the lowest rank k_2, s.t. p' has a mapping into l_q whose image starts at k_2 and ends *above* k_2', where if $k_2' = |l_q| + 1$ above means at $k_2' - 1$; if no such value exists, set k_2 to 0. Output the descriptor $\mathbf{pl}[\mathbf{f}, \mathbf{k_1}, (\mathbf{k_2}, \mathbf{l})]$.
   - otherwise, if there is no mapping of $l//p'$ into l_q whose image starts at $\text{ROOT}(l_q)$ and ends above k_2', add the descriptor $\mathbf{l}(\mathbf{S})$ to the set of view descriptors.

7.3.4 Computing m-descriptors

For this part, we need to check that some view v_j can be seen as being of the form *prefix*$_j$//m'//*suffix*$_j$, s. t. $m' \equiv m$ and

- *prefix*$_j$ root-maps into ft but $\text{MB}($*prefix*$_j)$ cannot root-map into $\text{MB}(ft)_m$,
- *suffix*$_j$ output-maps into lt, but $\text{MB}($*suffix*$_j)$ cannot output-map into $\text{MB}(lt)_m$.

Each of these aspects of an expansion is captured by a different type of fragment descriptor. We will output a view descriptor $\mathbf{m}(\mathbf{S})$ when a rule $f(X) \rightarrow l[\ldots]//g(X)$ is available and when (via fragment descriptors) we have that:

- g has an expansion v_g that gives us the part m'//*suffix*$_j$,
- there exist views generated via that rule and v_g, s.t. the part above v_g (in other words, the view obtained by expanding g in the empty pattern) has the properties for *prefix*$_j$.

We can use separate subroutines for each of these two items, and then the overall step above will combine their individual results.

For the *suffix*$_j$ part, we use *below m-descriptors*:

DEFINITION 7.5. Syntax: *For a unary fragment name f, a* below m-descriptors *is a tuple $bm[f, k_1, (k_2, p)]$, where k_1 and k_2 denote ranks in q, and p denotes any substring of* $\text{MB}(q)$.

Semantics: *There exists an expansion v_f s.t. (a) v_f contains the subtree of ft rooted at the node of rank k_1, (b) p is the main branch of the first token of v_f, and (c) k_2 is the start of the* lowest possible output-mapping image *of the main branch of the rest of v_f (besides p) into* $\text{MB}(lt)_m$*; by convention, k_2 is $|\text{MB}(q)| + 1$ when v_f has only one token and is* 0 *when there is no such mapping.*

Then, for the m part, we use *partial m-descriptors*:

DEFINITION 7.6. Syntax: *For a unary fragment name f, a* partial m-descriptor *is a tuple $pm(f, k)$, where k is a number in the range* 1 *to* $|\text{MB}(m)|$*, indicating a suffix of m.*

Semantics: *This descriptor says that (a) v_f is of the form m'//*suffix$_j$*, s.t. m' is equivalent with m's suffix having k main branch nodes, and (b)* suffix$_j$ *has the properties described above.*

For the *prefix*$_j$ part, we use *above m-descriptors*:

DEFINITION 7.7. Syntax: *For a unary fragment name f, an* above m-descriptor *is a tuple $am[f, k_1, (k_2, p)]$, where k_1, k_2 denote ranks in q and p is any substring of* $\text{MB}(q)$.

Semantics when p is empty (denoted hereafter '–')*: there exists an expansion v of the QSS s.t. (a) v is of the form $rest//v_f$, for v_f being an expansion of f (b) $rest$ root-maps into ft such that its image ends at the rank k_1, and (c) the end (bottommost node) of the* highest possible root-mapping *image of* $\text{MB}(rest)$ *into* $\text{MB}(ft)_m$ *is k_2; if no such mapping exists, by convention k_2 is* $|\text{MB}(ft)_m|+1$.

Semantics when $p \neq$ '–'*: there exists an expansion v of the QSS s.t. (a) v is of the form $rest//p'/v_f$, for $p = \text{MB}(p')$, (b) $rest//p'$ root-maps into ft such that the image of p' ends at the rank k_1, and (c) the end (bottommost node) of the* highest possible root-mapping *image of* $\text{MB}(rest)$ *into* $\text{MB}(ft)_m$ *is k_2; by convention, if no such mapping exists, k_2 is* $|\text{MB}(ft)_m| + 1$*; when $rest$ is empty k_2 is* 0.

Given a rule $f(X) \rightarrow l[\ldots]/g(X)$ or $f(X) \rightarrow l[\ldots]//g(X)$, we will use an am-descriptor for f to infer one for g.

Step 4 **of** findDescSupp(*q*, $\mathcal{P}$)**.**

Below m-descriptors are computed by a similar approach (one token at time) as the one used for partial l-descriptors. The *above m-descriptors* are obtained similarly, but in top-down manner. Starting from below-m descriptors, the *partial m-descriptors* are computed bottom-up, by simple equivalence checks.

If for some fragment name g we computed both an above m-descriptor $am[g, k_1, (|\text{MB}(ft)_m|+1, -)]$ and a partial m-descriptor $pm(g, |\text{MB}(m)|)$, we can add a descriptor $\mathbf{m}(\mathbf{S})$ to the set of view descriptors. (For more details we refer the reader to [7].)

We can now prove the following:

THEOREM 7.4. *Given a QSS $\mathcal{P}$ and a multi-token query q, algorithm* findDescSupp *is sound and complete for computing the descriptors for $\mathcal{P}$'s expansions.* findDescSupp *runs in polynomial time in the size of the query and of the QSS.*

By Theorems 7.4, 7.3 and 7.1, for a multi-token *XP* query q and QSS $\mathcal{P}$, given the descriptor set $\mathcal{D} :=$ findDescSupp$(q, \mathcal{P})$, q is supported by $\mathcal{P}$ if testEquivDesc(q, $\mathcal{D}$) outputs true.

Moreover, by Theorem 7.2, if q is in XP_{es}, it is supported by $\mathcal{P}$ (considering for now only rewrite plans that intersect views) if and only if testEquivDesc(q, $\mathcal{D}$) outputs true. We generalize these two observations to support in $XP^{\cap}$ in the next section.

7.4 Support with compensated views

We consider in this section general $XP^{\cap}$ rewrite plans for support that, before performing the intersection step, might compensate (some of) the views.

We show that support in this new setting can be reduced to support by rewrite plans which only intersect expansions of a QSS. This allows us to reuse the PTIME algorithms given in Section 7 (testEquivDesc and findDescSupp) and to find strictly more rewritings, namely those that would not be feasible without compensation. Thus we obtain a sound algorithm for support on *XP* multi-token queries in the rewrite language $XP^{\cap}$. This algorithm becomes complete when the input query is from XP_{es}.

Our reduction relies on the same QSS transformation, $comp(\mathcal{P}, q)$, used in Section 5.1, which builds expansions with compensation.

EXAMPLE 7.4. *Suppose that the QSS of the source in Example 3.1 is modified to return the guided trips themselves instead of the museums of those trips, by changing the third rule into rule* R_3*:*

$$(R_3): \quad f_1(X) \rightarrow trip(X)[guide].$$

and obtaining a new QSS $\mathcal{P}_2$*. Then, one of the expansions of* $\mathcal{P}_2$ *is:*

$$v_3\text{: doc(T)//vacation//trip/trip[guide]}$$

A query plan that rewrites q_2 *using compensated views is*

$$doc(v_3)/v_3/trip/museum \cap doc(v_2)/v_2/museum.$$

We can infer this rewriting by compensating R_3 *with a navigation to a museum child, which leads to a QSS identical to* $\mathcal{P}$*.*

We can prove the following:

THEOREM 7.5. *Given a QSS* $\mathcal{P}$ *and a multi-token XP query* q*, let* $\mathcal{D} :=$ findDescSupp$(q, comp(\mathcal{P}, q))$.

1. *Algorithm* testEquivDesc*(*q*,*$\mathcal{D}$*) is sound for support in* $XP^{\cap}$*, i.e.,* q *is supported by* $\mathcal{P}$ *in* $XP^{\cap}$ *if* testEquivDesc*(*q*,*$\mathcal{D}$*) outputs true.*
2. testEquivDesc*(*q*,*$\mathcal{D}$*) is also complete if* q *belongs to* XP_{es}*, i.e.* q *is supported by* $\mathcal{P}$ *in* $XP^{\cap}$ *iff* testEquivDesc*(*q*,*$\mathcal{D}$*) outputs true.*

Remark. In a setting in which one needs to also find a witness for support, this can be done by keeping at each step beside a descriptor one *representative*, an arbitrarily chosen view or view fragment from that equivalence class. More details can be found in [7].

8. SINGLE-TOKEN QUERIES

We consider in this section the remaining sub-fragment of XP_{es}, namely single-token queries. We show that id-based support becomes NP-hard (Theorem 8.1). Contrast this with both id-support for queries that have at least one //-edge in the main branch, and the rewriting problem for single-token XP_{es} queries under an explicit set of views, for which PTIME decision procedures exist.

THEOREM 8.1. *For an* XP_{es} *single-token query* q *and a QSS* $\mathcal{P}$*, deciding if* q *is supported by* $\mathcal{P}$ *in* $XP^{\cap}$ *is NP-hard.*

The surprising dichotomy between support for single-token and multi-token extended skeletons is rooted in their differences on the respective tests for equivalence with an intersection of views.

First, for the single-token case, it is easy to see that support can hold only if some view's main branch is equivalent to q's /-edges only main branch. Otherwise, one could easily exhibit interleavings that do have //-edges in their main branch, hence cannot be contained in q. With this, building interleavings amounts basically to deciding where to collapse main branch nodes from the various views on a linear path with /-edges only. Intuitively, it is now less a matter of how to order main branch nodes of the views, and more of choosing for each node a coalescing option among the few available. By consequence, a candidate interleaving i (i.e., one that is equivalent to q and contains all other interleavings) might combine (put under the same main branch node) predicates coming from different views *at all levels of the main branch*. When q has several tokens, this is true only for the candidate's first and last tokens (built by combining in the only way possible the first and last tokens of the views), while the section in between has to be entirely present (isomorphic modulo minimization) in some view.

The proof of Theorem 8.1 is given in [7]. We also give there an algorithm that decides support for XP_{es} single-token queries in exponential-time, i.e., the best we can hope for given the NP lower bound. Finally, we give a sound PTIME algorithm for this problem. For space reasons, and because the multiple-token queries are the more widely used class, further details on single-token queries are relegated to [7].

9. QSS WITH PARAMETERS

We consider now an extension to QSS with input parameters for text values (denoted $QSS^{\#}$) and correspondingly, an extension of *XP* to text conditions. We modify the grammar of *XP* as follows:

$$pred ::= \epsilon \mid [rpath] \mid [rpath = C] \mid [.//rpath] \mid [.//rpath = C] \mid pred\, pred$$

where C terminals stand for text constants. Every node in an XML tree t is now assumed to have a text value *text(t)*, possibly empty. The duality with tree patterns is maintained by associating to every predicate node n in a pattern p a test of equality *test(n)*, that is either the empty word or a constant C. The notions of embedding, mapping and containment can be adapted in straightforward manner to take into account text equality conditions.

The definition of $QSS^{\#}$ can be obtain from Definition 3.1 by adding the following: "a leaf element nodes may be additionally labeled with a parameterized equality predicate of the form $= \#i$, where $\#i$ is a *parameter* and i is an integer".

EXAMPLE 9.1. *Let us add to* $\mathcal{P}$ *from Example 3.1 the rule*

$$f_1(X) \rightarrow trip[maxprice = \#1]//museum(X)$$

Using this rule, we can generate the view v_4 *that retrieves museums on trips for which the maximum price is a parameter* $\#1$*:*

$$v_4\text{: doc(T)//vacation//trip/trip[maxprice=\#1]//museum}$$

A user query q_3 *that asks for museums with temporary exhibitions on secondary trips that cost at most* $1000

$$q_3\text{: doc(T)//vacation//trip/trip[maxprice=1000]//museum[temp]}$$

is then supported by the QSS, because it can be rewritten as

$$doc(v_4)/v_4/museum[temp](1000)$$

where parameter $\#1$ *is bound to the value in parenthesis* (1000).

We can show that all the tractability and hardness results presented in the previous sections remain valid when text conditions and parameters are added to the setting. Only minor adjustments are necessary in order to reuse the same PTIME algorithms for expressibility and support (modulo the new *XP* syntax and the adapted definitions of mapping and containment). Given a query q, the input $QSS^{\#}$ will be transformed into a QSS $\mathcal{P}'$ by replacing each $= \#i$ parameter occurrence by an explicit text equality condition $= C$, for each constant C appearing in q. Further details are omitted.

10. TRACTABILITY BOUNDARIES

We consider now extensions to the rewrite language and to the query set specifications, asking whether the efficient algorithms of the previous sections can be adapted to deal with them.

Compensated rewriting plans. We consider in this section more complex rewrite plans for support, beyond $XP^{\cap}$, taking the compensation idea one step further. More precisely, we consider the rewrite language $XP^{\cap,c}$ which, after the intersection step, might compensate again for equivalence with the input query. We capture $XP^{\cap,c}$ by adding the following rules to the grammar of *XP*:

$$\begin{array}{lll} ipath & ::= & cpath \mid (cpath) \mid (cpath)/rpath \mid (cpath)//rpath \\ cpath & ::= & apath \mid apath \cap cpath \end{array}$$

Revisiting Definition 2.6, a rewriting r in the language $XP^{\cap,c}$ is now of the form $\mathcal{I} = (\bigcap_{i,j} u_{ij})$, $\mathcal{I}/rpath$ or $\mathcal{I}//rpath$, with each u_{ij} being of the form $doc(v_j)/v_j/p_i$ or $doc(v_j)/v_j//p_i$.

EXAMPLE 10.1. *Consider the query q_4 below that extracts the temporary exhibitions from the data about museums visited on the same tour trips as in query q_1:*

q_4: doc(T)//vacation//trip/trip[guide]//tour[schedule//walk]/museum/temp

There is no rewriting of q_4 using only an intersection of views generated by $\mathcal{P}$, since there is no mention of temporary exhibitions in $\mathcal{P}$. However, if we allow the intersection to be compensated, q_4 can be rewritten as the intersection of v_1 and v_2, followed by a one-step navigation:

$$(doc(v_1)/v_1/museum \cap doc(v_2)/v_2/museum)/temp.$$

We prove that support in $XP^{\cap,c}$ becomes NP-hard even for multi-token XP_{es} queries:

THEOREM 10.1. *For a multi-token XP_{es} query q and a QSS $\mathcal{P}$, deciding if q is supported by $\mathcal{P}$ in $XP^{\cap,c}$ is NP-hard.*

The intuition behind this result is that an $XP^{\cap,c}$ rewriting r for a query q amounts to finding a rewriting r' in the simpler language $XP^{\cap}$ for a prefix of q and then compensating r' with the remainder of q. Even if q were multi-token, r' may correspond to a prefix of q that is in fact single-token, hence the complexity jump.

The proof of Theorem 10.1 is similar to the one of Theorem 8.1. In [7] we also show that support in $XP^{\cap,c}$ can be solved in exponential-time for input queries from XP_{es}, which is optimal for practical purposes. For space reasons, further details are omitted.

QSS with forest RHS. We consider now an extension to the query set specifications, which allows *forests* of tree fragments on the RHS, i.e., expansion rules of the form $f \rightarrow tf_1, \ldots, tf_k$.

We call the set specifications in this language QSS^+. With this added feature, we show that expressibility and support become NP-hard, even for very restricted tree patterns, without //-edges.

THEOREM 10.2. *Expressibility is NP-hard for QSS^+, even for XP queries and views without //-edges. Support is NP-hard for QSS^+, for XP queries and views without //-edges in predicates.*

We refer the reader to [7] for further details. There, we also show that expressibility can be solved in exponential time when views are encoded as a QSS^+.

11. RELATED WORK

XPath rewriting using only one view [17, 10] or a finite, explicitly given set of views [3, 2, 15, 5] was the object of several studies. To the best of our knowledge, we are the first to address the problem of rewriting XPath queries using a compactly specified set of views. The specifications are written in the Query Set Specification(QSS) language [14], which was also the basis for building a QBE-like XPath interface in a software system for managing biological data [12]. The QSS language presented in [14] has a different syntax from the one we adopted here and in [7] we show how that syntax can be compiled into ours.

Expressibility and support were studied in the past for relational queries and sets of relational views specified by Datalog programs [9, 16, 6]. The work on relational views [6] shares with our paper the idea of grouping the views in a finite number of equivalence classes w.r.t their behavior in a rewriting algorithm. Similar is also the strategy of computing these classes (represented by *descriptors*) bottom-up from the specification of the sets of views.

However, relational and XPath queries exhibit very different behaviors. For instance, support and expressibility were shown to be inter-reducible in PTIME for relational queries and views [6], and thus share the same complexity (EXPTIME-complete). This is no longer the case for *XP* queries in the presence of node Ids: expressibility is in PTIME (see Section 4), while support is coNP-hard. The PTIME results we obtain make crucial use of the tree shape of XPath queries and require problem-specific restrictions that do not follow from the relational work.

For implementing security policies, a complementary approach to specifying sets of views consists in annotating the DTD of the source with *access annotations* that can be used to allow/disallow access to parts of the data [8]. The system infers *one view* over the input document that conforms to the annotations and publishes the DTD of this view. Clients are allowed to ask any queries over the view DTD. This architecture is designed for security scenarios and does not extend to querying sources with limited capabilities.

12. CONCLUSION

We study the problems of expressibility and support of an XPath query by XPath views generated as expansions of a Query Set Specification. Since we focus on efficiency, we consider only PTIME algorithms, ensuring that they are sound in general and identifying the most permissive restrictions under which they become complete. We find that for XPaths corresponding to the fragment having child and descendant navigation and no wildcard, expressibility can be solved in PTIME. For support, the complexity analysis is more refined, as it depends on the rewriting language. In the case in which the XML nodes in the result of the views lose their original identity, we are able to give a PTIME algorithm for support. If the source exposes persistent node ids, which enable rewritings that intersect several views, we show that the problem becomes NP-hard unless fairly permissive restrictions on the user query are placed. We present a sound PTIME algorithm that also becomes complete under the restrictions.

13. REFERENCES

[1] S. Amer-Yahia, S. Cho, L. Lakshmanan, and D. Srivastava. Tree pattern query minimization. *VLDB J.*, 11(4), 2002.

[2] A. Arion, V. Benzaken, I. Manolescu, and Y. Papakonstantinou. Structured materialized views for XML queries. In *VLDB*, 2007.

[3] A. Balmin, F. Özcan, K. S. Beyer, R. Cochrane, and H. Pirahesh. A framework for using materialized XPath views in XML query processing. In *VLDB*, 2004.

[4] M. Benedikt, W. Fan, and G. Kuper. Structural properties of XPath fragments. *Theor. Comput. Sci.*, 336(1), 2005.

[5] B. Cautis, A. Deutsch, and N. Onose. XPath rewriting using multiple views: Achieving completeness and efficiency. In *WebDB*, 2008.

[6] B. Cautis, A. Deutsch, and N. Onose. Querying data sources that export infinite sets of views. In *ICDT*, 2009.

[7] B. Cautis, A. Deutsch, N. Onose, and V. Vassalos. Efficient rewriting of XPath queries using Query Set Specifications, 2009. TR CS2009-0941, UCSD. Available from http://db.ucsd.edu/index.jsp?pageStr=publications.

[8] W. Fan, C. Y. Chan, and M. N. Garofalakis. Secure XML querying with security views. In *SIGMOD Conference*, pages 587–598, 2004.

[9] A. Y. Levy, A. Rajaraman, and J. D. Ullman. Answering queries using limited external query processors. *JCSS*, 58(1), 1999.

[10] B. Mandhani and D. Suciu. Query caching and view selection for XML databases. In *VLDB*, 2005.

[11] G. Miklau and D. Suciu. Containment and equivalence for a fragment of XPath. *J. ACM*, 51(1), 2004.

[12] S. Newman and Z. M. Özsoyoglu. A tree-structured query interface for querying semi-structured data. In *SSDBM*, pages 127–130, 2004.

[13] Y. Papakonstantinou, A. Gupta, H. Garcia-Molina, and J. D. Ullman. A query translation scheme for rapid implementation of wrappers. In *DOOD*, 1995.

[14] M. Petropoulos, A. Deutsch, and Y. Papakonstantinou. The Query Set Specification Language (QSSL). In *WebDB*, pages 99–104, 2003.

[15] N. Tang, J. Yu, T. Özsu, B. Choi, and K. Wong. Multiple materialized view selection for XPath query rewriting. In *ICDE*, 2008.

[16] V. Vassalos and Y. Papakonstantinou. Expressive capabilities description languages and query rewriting algorithms. *J. Log. Program.*, 43(1), 2000.

[17] W. Xu and Z. M. Özsoyoglu. Rewriting XPath queries using materialized views. In *VLDB*, 2005.