A Very Accurate Approximation for Cell Loss Ratio in ATM Networks[♣]

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Abstract

We like to find the CLR in ATM networks when the statistical multiplexing is an important factor. In this paper, first we have proposed the combination of three analytical expressions, which approximate the cell loss probability, based on the fluid-flow approximation model and two stationary approximation models. Second, we have provided a very accurate numerical model for the finite buffer, which lies at the input of each VP. The sources are statistically independent and each traffic source has a two-state Markov model. This simulation is done at the cell level and its results are very accurate. We have compared the results of the numerical simulation with the results of the analytical approximation models. Also we have used the linear estimation to find an accurate expression for cell loss approximation in ATM networks

1. Introduction

ATM as a high-speed cell switching technology can support multiple classes of traffic sources with different quality of service (QoS) requirements and diverse traffic characteristics. In this study we are interested in one of the QoS requirements: cell loss probability.

In our study the sources are statistically independent and each traffic source has a two-state Markov model [1]. A single source has a variable bit rate alternated asynchronously between *On* and *Off* state and bounded by the peak rate r. Such a source in an *On* state transmits at peak rate and in an *Off* state transmits at zero bit rate. The duration of the *On* and the *Off* state are assumed to be exponentially distributed and therefore the source is completely characterized by three parameters, namely peak rate r, utilization ρ, and b, where ρ is the fraction of time the source is active and b is the mean of the *On* state period. Other parameters of interest, such as the mean m and the variance σ^2 of the bit rate are identified completely from the source metric vector (r, ρ, b):

$$m = \rho \, . \, r$$

and $$\sigma^2 = \rho\,(1-\rho)\, r^2$$

The advantages of the above physical model are its simplicity and flexibility, such as it can be used for connections ranging from burst to continuous bit streams.

The remainder of this paper is organized as follows: in section 2, we discussed analytical approximation models. In section 3, we proposed an accurate numerical model for finding the cell loss probability in the finite buffer, which lies at the input of each VP. In section 4, we

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proposed a new accurate expression for the cell loss ratio in the buffers of ATM switchs. The conclusion of our study is discussed in section 5.

2. Combination of Three Models

In this paper, first we have proposed the combination of three analytical expressions, which approximate the cell loss probability, based on the fluid-flow approximation model and two stationary approximation models. These models have been proposed to approximate the equivalent capacity of two-state Markov sources. Most of researchers that studied the routing in ATM networks used only the results of the fluid-flow approximation model for the call admission function ([5], [6], [7], [8]). But, we showed that for a good approximation, we must combine all the existing models to obtain an accurate result for different ranges of connections characteristics. The following expressions calculate the cell loss ratio in a finite buffer, which lies at the input of each VP. We consider a finite buffer with the capacity of x (Mbit) capacity, FIFO queuing and two-state Markov (*On-Off*) arrival traffic. Let F be the ratio of the VP capacity C to the VC peak rate r ($F=C/r$) and L be the number of VCs in the VP. Considering to the fluid-flow approximation model[1],[2], we have:

$$P1_{loss}(L) \begin{cases} = \exp\left(-\frac{x}{r}\left(1+\delta-\frac{L\delta}{F}\right)/\left(1-\frac{F}{L}\right)\right) & \text{if}\ (L>F) \\ = 0 \quad \text{if}\ (L\leq F) \end{cases} \tag{1}$$

On the other hand, we have found $P2_{loss}(L)$, which is the cell loss probability obtained from stationary approximation using binomial distribution. We have:

$$F = k' = \left[\frac{C}{r}\right] \quad , \quad P_k = \binom{L}{k} \rho^k (1-\rho)^{L-k}$$

$$P2_{loss}(L) = \sum_{k=F+1}^{L} P_k \tag{2}$$

Also, we have obtained $P3_{loss}(L)$, which is the cell loss probability based on stationary approximation using Gaussian distribution, as follows:

$$P3_{loss}(L) = e^{\frac{r^2}{L}\left(-\frac{(F-\rho L)^2}{2L\sigma^2}\right) - 0.5\ln(2\pi)} \tag{3}$$

Since, all three of the above approximations are conservative and valid upper bounds[2],[8] (we will show this fact according to our numerical results), $P_{loss}(L)$ can be obtained from the following expression:

$$P_{loss}(L)=\min\{P1_{loss}(L), P2_{loss}(L), P3_{loss}(L)\} \tag{4}$$

3. A Numerical Model

We like to provide a very accurate numerical model for the finite buffer, which lies at the input of each VP. Just like the analytical models, here again, we consider a finite buffer with the capacity of x (Mbit) capacity, FIFO queuing and two-state Markov (*On-Off*) arrival traffic. The result, which has to be calculated at the end, is the buffer overflow probability (the loss probability). Figures 1 show this model briefly. We will obtain the P_{loss} for different values of L and δ by providing a program in C++ language based on this model. The aims and objectives of this simulation are as follows:

- This simulation is done at the cell level and the results of the numerical model are very important. Figures 2 to 4 show the results of our simulation.
- Finding an accurate expression for cell loss ratio is the main aim of this research.
- Evaluation of different analytical methods, which have represented in this paper for P_{loss} approximation. In this research we have compared the results of the numerical simulation with the results of the Fluid-flow approximation, stationary approximation using Gaussian distribution and stationary approximation using binomial distribution methods.
- Proving the fact that the analytical methods of the P_{loss} approximation are inaccurate and conservative and each of them works out better in a particular range of L (the number of VCs in the VP) and δ ($1/\delta$ is the mean of *Off* periods). These results lead to the substantiation of the expression, which has obtained from the combination of all the three analytical approximation methods.
- The result of the simulation will help us to determine the minimum, maximum and average error of each of the analytical methods. These results lead to find out the more accurate expressions for calculating the cell loss probability.
- This model has a great flexibility, such that we can also use it for other disciplines (other than FIFO) such as weighted fair queuing (WFQ), just by modifying some variables and a small part of the program logic. Also, we can easily use an arrival traffic model else than *On-Off* Markov model by modifying the random generators (as an instance, we can use the self-similar traffic generator). These changes are not feasible easily in analytical models and all the algebraic calculations must be repeated from the beginning or we have to ignore the model entirely and using other analytical models. We can use non-homogeneous traffics in this model, too.

Considering to the simulation results, if we assume that the maximum error coefficient (that is equivalent to the minimum of the proportion of the analytical and numerical results) in $P1_{loss}$, $P2_{loss}$, and $P3_{loss}$ are respectively $\alpha 1$, $\alpha 2$ and $\alpha 3$, then we can write the previous analytical expressions (1, 2, and 3) in a more accurate form as follows:

$$P1_{loss} = \alpha 1 * P1_{loss}\ (\text{old})$$
$$P2_{loss} = \alpha 2 * P2_{loss}\ (\text{old})$$
$$P3_{loss} = \alpha 3 * P3_{loss}\ (\text{old})$$

Where:

$$\alpha 1 = 1.04 * 10^{-2}$$
$$\alpha 2 = 9.55 * 10^{-3}$$
$$\alpha 3 = 7.37 * 10^{-3}$$

Finally, $P_{loss}(L)$ can be calculated from the following expression:

$$P_{loss}(L) = \min\{P1_{loss}(L),\ P2_{loss}(L),\ P3_{loss}(L)\} \quad (5)$$

To find the upper bound for P_{loss} , we used the maximum error coefficient. These expressions are the results of combining the analytical and numerical methods, which are discussed in this paper.

We repeated the simulation for other values of the VP capacity and the buffer size and we saw that always the expression (5) is a valid upper bound for P_{loss}.

4. A Very Accurate Model

Although the results of the numerical model are very accurate, but these results can not be used directly in the ATM routing algorithm or call admission function. Actually we need an explicit and simple expression, which can approximate P_{loss} as a function of different parameters such as L, δ, x, and F(C/r). In the previous section, we found constant coefficients for compensating the error of the P_{loss} expression, which have been obtained by the three analytical models. But in this section, we like to estimate a new accurate expression for cell loss ratio based on the results of the exact numerical model. In other words, we want to find the P_{loss} expression as a function of the model parameters. Usually the desired cell loss probability is considered in the range of 10^{-6} to 10^{-9} [2]. But we have considered the range of 10^{-3} to 10^{-9} for P_{loss} to find a valid expression in a wide range of L. We will try to find an expression, which can approximate P_{loss} in this range with high accuracy.

The figures 2 to 4 show PN_{loss} (the numerical model results) in comparison with the results of the three analytical approximation models, for δ respectively equal to 0.125, 1, and 5. Note that these curves are drawn logarithmically. We have chosen these values (0.125, 1 and 5) for δ to find a general expression which can be used in a wide range of arrival traffics, from almost burst traffic (δ=0.125, ρ=1/9) to nearly continuous bit streams (δ=5, ρ=5/6).

Figures 2 to 4 show that the logarithm of PN_{loss} in the range of 10^{-3} to 10^{-9} is a linear function of L. So, we can use the linear estimation method to obtain an expression for the P_{loss}. But the problem is that each of these lines can be used just for particular F (F=C/r), x and δ. In the other word, if F, x and δ are constant and the P_{loss} (we have called it ψ) is a function of L (Number of VCs), then the slope of ψ will be a function of F, x and δ. We have the following expressions:

$$\ln(P_{loss})=\psi(L, F, x, \delta)$$

Since ρ=δ/(δ+1), we can write:

$$\ln(P_{loss})=\psi(L, F, x, \rho)$$

and

$$\partial\psi(L, F, x, \rho) / \partial L = f(F, x, \rho)$$

Considering that ψ is a linear function of L, Table 1 shows the end points of 27 lines, which estimate ψ for x of 24, 48, and 96, F of 25, 50, and 100 , and δ of 0.125, 1, and 5. These ranges are considered wide to achieve an expression, which is valid in all ranges of F, x, L, and δ. We have liked to find a line, in which the end points ((X1,Y1) and (X2,Y2)) are the function of F, x, and ρ. These parametric end points must be fit to all of 27 end points ((x1,y1) and (x2,y2)) in Table 1.

As we have already mentioned, in ATM networks, the desired CLR is considered in the range of 10^{-6} to 10^{-9} [2]. But we have considered the range of nearly 10^{-3} to nearly 10^{-9} (because of limitation of the results of simulation, these values are not exactly equal to 10^{-3} and 10^{-9}) for P_{loss} to find a valid expression in a wide range of L. We will try to find an expression, which can approximate P_{loss} in this range with high accuracy. Since in all end points, y1 is nearly equal to $9.5*10^{-8}$ and y2 is nearly equal to $1.9*10^{-3}$ (these values are the averages of y1 and y2 in Table 1), we can write:

$$Y1=\ln(9.5*10^{-8})=-16.17$$
$$Y2=\ln(1.9*10^{-3})=-6.27$$

Although Y1 and Y2 found easily, but finding X1 and X2 as functions of network and traffic parameters are very hard. We have guessed each of terms of the following expressions individually. Then we have write a program to find the coefficients of the expressions by try end error (iteration) method. Finally we have found the following expressions, which are the accurate approximation of X1 and X2 :

$$X1=0.53\frac{F}{\rho}+0.5F+\frac{5}{\delta}(\frac{F}{25}-1)+\frac{2.5}{\delta}(\frac{x}{24}-1)$$
$$X2=0.8\frac{F}{\rho}+0.24F+\frac{2}{\delta}(\frac{F}{25}-1)+\frac{2.5}{\delta}(\frac{x}{24}-1)$$

The linear estimation can be written as Follows:

$$y=\frac{Y2(X1-x)+Y1(x-X2)}{X1-X2}$$

After replacement of X1 and X2 expressions and Y1 and Y2 values in the above line function and a simple modification, we can write:

$$Ploss=e^{\frac{171.6-64.35F-6.875x+66L\delta-69.058F\delta}{20+F+0.066F\delta}} \quad (6)$$

So, we can use the expression (6) for calculating P_{loss} in the routing algorithms of ATM networks.

5. Conclusion

In this paper, first we discussed three analytical approximation methods for cell loss ratio and combined these methods for finding the more accurate expression (4). Since the results of the P_{loss} expression (4) are not accurate, we provided a very accurate numerical model for the finite buffer at the input of each VP. We used the maximum error coefficient and found a more accurate expression (5) for P_{loss}. Then we used the linear estimation to find the P_{loss} as a function of the model parameters and found a very accurate expression for calculating the cell loss probability (expression (6)).

The curves of figures 2 to 4 show that our approximation model ($P6_{loss}$, which can be calculated by expression (6)) is accurate (note that PN_{loss} is the results of an accurate model of P_{loss}).

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Table 1. The End Points of Estimated lines

x	F	δ	ρ	x1	y1	x2	y2
24	25	0.125	1/9	130	$1.15*10^{-7}$	186	$1.62*10^{-3}$
		1	0.5	39	$1.34*10^{-7}$	47	$2.01*10^{-3}$
		5	5/6	29	$1.92*10^{-8}$	30	$3.05*10^{-3}$
	50	0.125	1/9	300	$1.24*10^{-7}$	390	$1.56*10^{-3}$
		1	0.5	82	$1.40*10^{-7}$	94	$1.34*10^{-3}$
		5	5/6	58	$4.00*10^{-8}$	60	$2.94*10^{-3}$
	100	0.125	1/9	641	$9.01*10^{-8}$	796	$2.18*10^{-3}$
		1	0.5	172	$1.38*10^{-7}$	191	$1.34*10^{-3}$
		5	5/6	116	$2.78*10^{-8}$	120	$2.84*10^{-3}$
48	25	0.125	1/9	150	$9.59*10^{-8}$	206	$2.29*10^{-3}$
		1	0.5	42	$9.90*10^{-8}$	49	$1.96*10^{-3}$
		5	5/6	29	$1.05*10^{-8}$	31	$2.06*10^{-3}$
	50	0.125	1/9	320	$1.87*10^{-7}$	405	$1.35*10^{-3}$
		1	0.5	86	$1.29*10^{-7}$	98	$1.29*10^{-3}$
		5	5/6	58	$1.00*10^{-10}$	60	$1.64*10^{-3}$
	100	0.125	1/9	665	$1.15*10^{-7}$	815	$1.23*10^{-3}$
		1	0.5	174	$1.29*10^{-7}$	194	$1.84*10^{-3}$
		5	5/6	117	$5.1*10^{-8}$	122	$1.11*10^{-3}$
96	25	0.125	1/9	192	$1.25*10^{-7}$	248	$1.93*10^{-3}$
		1	0.5	47	$1.05*10^{-7}$	54	$1.74*10^{-3}$
		5	5/6	30	$9.67*10^{-8}$	32	$3.08*10^{-3}$
	50	0.125	1/9	364	$1.83*10^{-7}$	446	$2.69*10^{-3}$
		1	0.5	91	$1.01*10^{-7}$	102	$1.01*10^{-3}$
		5	5/6	60	$7.5*10^{-8}$	62	$1.07*10^{-3}$
	100	0.125	1/9	705	$8.33*10^{-8}$	854	$2.62*10^{-3}$
		1	0.5	180	$9.99*10^{-8}$	198	$2.21*10^{-3}$
		5	5/6	118	$6.1*10^{-8}$	123	$1.31*10^{-3}$

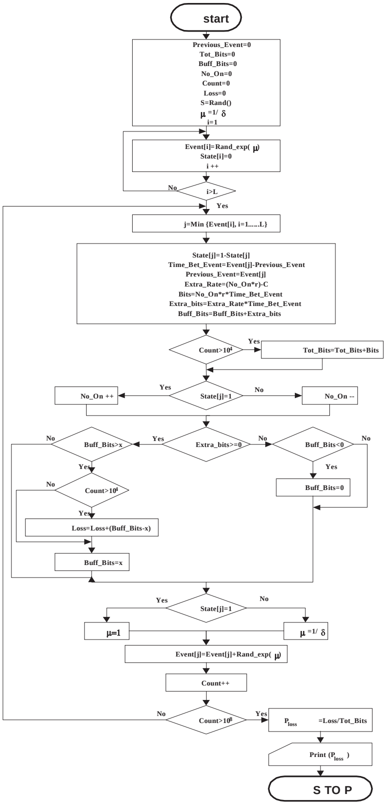

Figure 1. Numerical Model of Finite Buffer with On_Off Markov Sources.

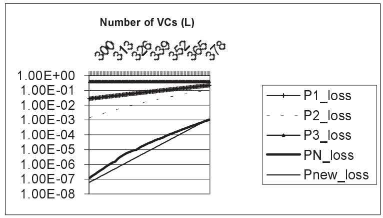

Figure 2. Comparison of the numerical results with the expressions 1, 2, 3, and 4 for x=24, F=50 and δ=0.125

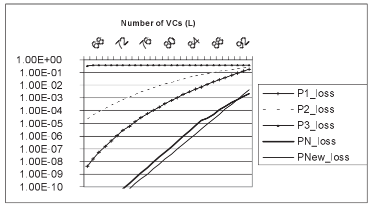

Figure 3. Comparison of the numerical results with the expressions 1, 2, 3, and 4 for x=24, F=50 and δ=1.0

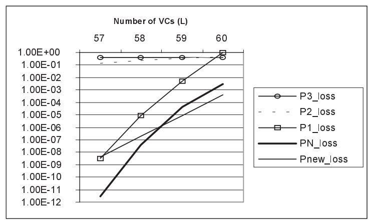

Figure 4. Comparison of the numerical results with the expressions 1, 2, 3, and 4 for x=24, F=50 and δ=5.0