A counter-example to Voloshin's hypergraph co-perfectness conjecture

Daniel Král'

Department of Applied Mathematics and
Institute for Theoretical Computer Science (ITI)*
Charles University
Malostranské náměstí 25, 118 00 Prague
Czech Republic
kral@kam.mff.cuni.cz

Abstract

The upper chromatic number $\overline{\chi}(H)$ of a hypergraph H is the maximum number of colors in a coloring avoiding a polychromatic edge. The stability number $\alpha(H)$ of a hypergraph H is the cardinality of the largest set of vertices of H which does not contain an edge. A hypergraph is k-uniform if the sizes of all its edges are k. A hypergraph H is co-perfect if $\overline{\chi}(H') = \alpha(H')$ for each induced subhypergraph H' of H.

Voloshin conjectured that an r-uniform hypergraph H ($r \geq 3$) is co-perfect if and only if it contains neither of two special r-uniform hypergraphs (a so-called monostar and a complete circular r-uniform hypergraph on $2r-1$ vertices) as an induced subhypergraph. We disprove this conjecture for all r.

1 Introduction

A *hypergraph* H is a pair (V, E) where V is its vertex set and $E \subseteq 2^V$ is its edge set; we do not restrict the sizes of the edges to two as in the case of graphs. Throughout the paper we write $V(H)$ for a vertex set of a hypergraph H and $E(H)$ for its edge set. Recently, the topic of coloring of vertices of hypergraphs avoiding a *polychromatic* edge (i.e., an edge whose vertices have mutually different colors) has drawn the attention of different researchers, cf. [4, 6, 10, 15, 16], and related extremal (anti-Ramsey) questions were studied in [1, 3, 7, 17]. In this case, we want to color a hypergraph with a maximum possible number of colors (coloring all the vertices with the same color is clearly good and thus minimizing the number of colors is not

* Institute for Theoretical Computer Science is supported by the Ministry of Education of Czech Republic as project LN00A056.

interesting); the maximum possible number $\overline{\chi}(H)$ of colors such that the vertices of the hypergraph H can be colored avoiding a polychromatic edge is called the *upper chromatic number* of H. Besides studying this type of coloring, the researchers also study so-called *mixed hypergraphs* where the coloring has to prevent some of the edges being monochromatic and some of them being polychromatic, cf. [5, 9, 11, 12, 13] and a recent monograph on the subject [18].

In this paper we study the coloring of hypergraphs which avoids a polychromatic edge as described in the previous paragraph. The *stability number* $\alpha(H)$ of a hypergraph H is the cardinality of the largest set A such that no edge of H is contained in A; such a set A is called *stable*. If c is a coloring of the vertices of H, then a *color class* with respect to c is a set of the vertices of H colored with the same color. It is clear that $\alpha(H) \geq \overline{\chi}(H)$, since we can create a stable set by taking one vertex from each color class of a coloring using $\overline{\chi}(H)$ colors (and this is actually a stable set, since the coloring avoids a polychromatic edge). A natural question is: "For which hypergraphs H does it hold that $\alpha(H) = \overline{\chi}(H)$?" A conjecture on this problem with the strong perfect graph conjecture favor was made in [19]: They conjectured that $\alpha(H') = \overline{\chi}(H')$ for all induced subhypergraphs H' of H if and only if H contains neither of two special types of hypergraphs as an induced subhypergraph.

A hypergraph H is *r-uniform* if the size of all its edges is r; a hypergraph H is *r-regular* if each of its vertices is contained in precisely r edges of H. A *subhypergraph* H' of a hypergraph H is a hypergraph whose vertex and edge sets are respectively subsets of the vertex set and the edge set of H; a subhypergraph H' is *induced* if $E(H') = E(H) \cap 2^{V(H')}$, i.e., all the edges of H all of whose vertices are in $V(H')$ are also the edges of H'. A famous strong perfect graph conjecture asserts that $\alpha(G') = \chi(G')$ for each induced subgraph G' of G (such graphs are called *perfect*) if and only if neither G nor its complement contains an odd cycle on 5 or more vertices as an induced subgraph. Voloshin, inspired by this famous conjecture, made a similar conjecture in [19]:

Conjecture 1 [19] *For each $r \geq 3$, an r-uniform hypergraph H is co-perfect if and only if it contains neither a monostar nor a C^r_{2r-1} as an induced subhypergraph.*

We postpone the missing definitions to the next paragraph. The co-perfectness of hypergraphs has been introduced in [19]; Conjecture 1 can be found as Conjecture 1 in [19]; the other conjecture stated in [19], Conjecture 2 of [19] on spectra of mixed hypergraphs, has been recently answered in the affirmative by the author in [9]. Other problems posed in [19] have been considered in [2, 8, 14] (Problem 8 of [19] on mixed hypergraphs derived from planar hypergraphs), in [9] (Problem 10 and Problem 11 of [19] about spectra of mixed hypergraph and some related extremal questions), in [12] (Problem 13 of [19] regarding an edge version of mixed hypergraphs) and in [11, 13] (Problem 14 of [19] about mixed hypergraphs with a restricted edge structure).

A hypergraph H is *co-perfect* if for each of its induced subhypergraph H' it holds that $\alpha(H') = \overline{\chi}(H')$. A *monostar* is a hypergraph H such that the cardinality of the intersection of all the edges of H is exactly one, i.e., there exists a vertex v which is contained in all the edges and v is a unique vertex with this property; we call such a vertex the *center vertex* of a monostar. It is clear that $\alpha(H) = n - 1$ for a monostar H on n vertices and $\overline{\chi}(H) < n-1$; hence monostars are certainly not co-perfect. A hypergraph H is *circular* if there exists a cycle (in the usual graph theory sense) on the vertices of H such that the edges of H form its paths; we write C_n^r for an r-uniform hypergraph whose edges are precisely all the paths consisting of r vertices of the n-vertex cycle, i.e., C_n^r is the maximum r-uniform circular hypergraph on n vertices. The hypergraph C_n^r for $n \geq 2r$ contains a monostar as an induced subhypergraph and thus it is not co-perfect; but also C_{2r-1}^r is not co-perfect, since $\alpha(C_{2r-1}^r) = 2r-3$ and $\overline{\chi}(C_{2r-1}^r) < 2r-3$ (cf. [19]). These two examples of non-co-perfect hypergraphs led to Conjecture 1 which is similar to the strong perfect graph conjecture, but besides this similarity there is no other connection between these two conjectures. Conjecture 1 has attracted attention of researchers: Tuza discussed Conjecture 1 during his invited talk on mixed hypergraphs at the Workshop, Cycles and Colorings 2001, in Stara Lesna, Slovakia. There is also a special chapter devoted to the concept of co-perfectness and to this conjecture in a recent monograph [18]. We provide a counter-example to this conjecture for any $r \geq 3$.

Conjecture 1 is clearly equivalent to the following conjecture:

Conjecture 2 *If an r-uniform hypergraph H ($r \geq 3$) contains neither a monostar nor C_{2r-1}^r as an induced subhypergraph, then $\alpha(H) = \overline{\chi}(H)$.*

Due to Conjecture 2, in order to disprove Conjecture 1, it is enough to find an r-uniform hypergraph H (for each $r \geq 3$) which contains neither a monostar nor C_{2r-1}^r as an induced subhypergraph and for which $\overline{\chi}(H) < \alpha(H)$. We prove the existence of such hypergraphs in Theorem 1 in Section 2.

1.1 Definitions and Notation

Let H be a hypergraph. We write $H \setminus V_0$ where $V_0 \subseteq V(H)$ for the induced subhypergraph of H on the vertex set $V(H) \setminus V_0$. Let c be a coloring of the vertices of H. If H contains no polychromatic edge, we say that the coloring c is *good*. A color of a vertex v is *unique* if v is the only vertex colored with this color. An *isomorphism* between two hypergraphs H_1 and H_2 is a one-to-one mapping $\varphi : V(H_1) \to V(H_2)$ such that the images of the edges of H_1 are precisely the edges of H_2. An isomorphism is an *automorphism* if $H_1 = H_2$; an automorphism is *non-trivial* if it is not the identity. A hypergraph H is *vertex-transitive* if for any two vertices v and w of H there is an automorphism φ of H such that $\varphi(v) = w$.

The *incidence matrix* of a hypergraph H with $V(H) = \{v_1, \ldots, v_n\}$ and $E(H) = \{e_1, \ldots, e_m\}$ is $n \times m$ matrix $I(H)$ such that $I(H)_{ij} = 1$ if $v_i \in e_j$ and $I(H)_{ij} = 0$ otherwise. Note that if H is r-uniform, then each column sum is precisely r; if H is k-regular, then each row sum is precisely k. We deal with different uniform hypergraphs in the paper: We use the notation such that the superscript is equal to the common sizes of edges, e.g., C_n^r (defined earlier) is an r-uniform hypergraph.

2 The Counter-Example

We first define the counter-example (to Conjecture 2) r-uniform hypergraph H^r:

Definition 1 *Let $r \geq 3$ be a fixed integer. Let H^r be the r-uniform hypergraph with $2r$ vertices and $2r+2$ edges whose incidence matrix $2r \times (2r+2)$ is the following (the incidence matrices for $r = 3$ and $r = 4$ can be found below):*

$$I(H^r) = \left(\begin{array}{cccccccccccc|cc}
1 & 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 1 & 0 & 0 & 1 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 1 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 & 1 & 1 & 0 & 1
\end{array}\right)$$

We write $v_1, \ldots, v_{2r}$ for the vertices of H^r; the vertex v_i corresponds to the i-th row of the incidence matrix. We write $e_1, \ldots, e_{2r}$ for the edges corresponding to the first $2r$ columns of the incidence matrix:

$$e_i = \{v_i\} \cup \{v_{i+1}, v_{i+3}, \ldots, v_{i+2r-3}\}$$

where the subscripts of the vertices are taken modulo $2r$. We write e_o and e_e (odd and even, corresponding to the parity of the indices of the subscripts of the vertices contained in e_o and e_e) for the edges corresponding to the last but one and the last column of the incidence matrix.

In order to illustrate the definition, we include the incidence matrices for H^3 and H^4:

$$I(H^3) = \left(\begin{array}{cccccc|cc}
1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 1
\end{array}\right)$$

$$I(H^4) = \left(\begin{array}{cccccccc|cc}
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1
\end{array}\right)$$

The main theorem of this section will be the following:

Theorem 1 *The r-uniform hypergraph H^r for any $r \geq 3$ contains neither a monostar nor the complete circular hypergraph C^r_{2r-1} on $2r-1$ vertices, but $\overline{\chi}(H^r) < \alpha(H^r)$.*

First, we make a simple observation on the structure of H^r:

Lemma 1 *The hypergraph H^r is a vertex-transitive r-uniform $(r+1)$-regular hypergraph.*

Proof: The proof of the uniformity and the regularity of H^r follows immediately from Definition 1. In order to prove the vertex-transitivity of H^r, note that the function $\varphi : V(H^r) \to V(H^r)$ defined as follows is an automorphism of H^r:

$$\varphi(v_i) = \begin{cases} v_1 & \text{if } i = 2r, \\ v_{i+1} & \text{otherwise.} \end{cases}$$

■

We next find the stability number of H^r:

Lemma 2 *The stability number of H^r is $2r-3$.*

Proof: The set of vertices of H^r, $\{v_1, \ldots, v_{2r-3}\}$, is stable; thus $\alpha(H^r) \geq 2r-3$. Let $A \subseteq V(H^r)$ be a stable set of size $2r-2$. We can assume that $v_1 \notin A$ because H^r is vertex-transitive. Let v_i be the only vertex different from v_1 not contained in A. If i is odd, then $e_e \subseteq A$. Hence i has to be even. If $i = 2$, then $e_3 \subseteq A$; but if $i \geq 4$, then $e_2 \subseteq A$. Hence A is not stable.

■

We next prove that the upper chromatic number of H^r is smaller than its stability number:

Lemma 3 *The upper chromatic number of H^r is $2r-4$.*

Proof: Let c be the following coloring of H^r:

$$c(v_i) = \begin{cases} i & \text{for } 1 \le i \le 2r-5, \\ 2r-4 & \text{for } 2r-4 \le i \le 2r. \end{cases}$$

The coloring c is a good coloring of H^r and thus $\overline{\chi}(H^r) \geq 2r-4$. In the rest, we prove that $\overline{\chi}(H^r) \leq 2r-4$. Let c be a coloring of H^r using $2r-3$ colors. We distinguish several cases to prove that this coloring is not good:

- There are four vertices sharing the same color (the sizes of the color classes are $4:1:1:\ldots:1$ in this case).
  Let A be the set of the four vertices colored with the same color. It has to be that $|A \cap e_o| = 2$ and $|A \cap e_e| = 2$ because e_o and e_e are disjoint. We may assume without loss of generality that $v_1 \in A$. Consider the edge e_2: There is only one vertex among the vertices $v_3, v_5, \ldots, v_{2r-1}$ from the set A (there are exactly two vertices with an odd index in A and one of them is v_1). Then v_2 must be in A. In a similar way, one may conclude that $v_3 \in A$, $v_4 \in A$ and $v_5 \in A$. This contradicts $|A| = 4$.

- There are exactly three vertices sharing the same color (the sizes of the color classes are $3:2:1:1:\ldots:1$).
  Let A be the set of the three vertices sharing the same color and B the set of the two vertices; set $S = A \cup B$. We may assume without loss of generality that $|S \cap e_o| = 2$ and $|S \cap e_e| = 3$. Let $\{v_i, v_j\} = S \cap e_o$. Consider the edge e_{i+1}: It has to be that $|S \cap e_{i+1}| \geq 2$ and this is possible only if $v_{i+1} \in S$; actually, the vertices v_j and v_{i+1} must have the same color. Similarly, $v_{j+1} \in S$ and the vertices v_i and v_{j+1} have the same color. But then there are four vertices of the same color which is impossible.

- There are no three vertices sharing the same color (the sizes of the color classes are $2:2:2:1:1:\ldots:1$).
  Let A, B and C be the three pairs of the vertices colored with the same color. We may assume without loss of generality that $A \subseteq e_o$ and $B \subseteq e_e$. Consider a vertex $v_i \in A$. Since the edge e_{i+1} does not contain v_i and its intersections with both A and B have sizes at most one, it has to be that $C \subseteq e_{i+1}$. Similarly $C \subseteq e_{j+1}$ for the other vertex $v_j \in A$ and $C \subseteq e_{k+1}$ and $C \subseteq e_{l+1}$ for the two vertices v_k and v_l from B (indices are taken modulo $2r$ if necessary). Then $C \subseteq e_{i+1} \cap e_{j+1} \subseteq e_o$ and $C \subseteq e_{k+1} \cap e_{l+1} \subseteq e_e$ which is impossible because e_o and e_e are disjoint.

■

It remains to check that H^r contains neither a monostar nor C^r_{2r-1} as an induced subhypergraph:

Lemma 4 *The hypergraph H^r does not contain a monostar as an induced subhypergraph.*

Proof: Let $r \geq 3$ be a fixed integer throughout the proof. We assume that H^r contains a monostar with the center vertex equal to v_1. Let $V_0 \subseteq V(H^r)$ be the vertices which induce the monostar and let $E_0 = E(H^r) \cap 2^{V_0}$. Note that the following hold, due to the definition of a monostar and an induced subhypergraph:

$$V_0 = \bigcup_{e \in E_0} e$$

$$\forall e' \in E(H) : e' \subseteq V_0 \Rightarrow e' \in E_0$$

$$\{v_1\} = \bigcap_{e \in E_0} e = \bigcap_{e \subseteq V_0, e \in E(H')} e$$

We distinguish several cases in the proof:

- $e_1 \in E_0$ and $e_o \in E_0$
  It has to be that $V_0 \supseteq e_o \cup e_1 = V(H^r) \setminus \{v_{2r}\}$. But then $e_2 \in E_0$ — contradiction.

- $e_1 \in E_0$ and $e_o \notin E_0$
  The edge e_1 cannot be the only edge of E_0. Since the intersection of the edges of E_0 is $\{v_1\}$, E_0 can contain, besides e_1, only the edges e_i for even $i, 4 \leq i \leq 2r$. If $e_{2r} \in E_0$, then $e_1 \cup e_{2r} = V(H^r) \setminus \{v_{2r-1}\} \supseteq V_0$ and $e_e \in E_0$ which is impossible. Let e_{i_1} be an edge of E_0 different from e_1; i_1 has to be an even integer between 4 and $2r-2$. Since $e_1 \cap e_{i_1} = \{v_1, v_{i_1}\}$, the edge set E_0 has to contain an edge e_{i_2} different from e_1 and e_{i_1}. But then $e_1 \cup e_{i_1} \cup e_{i_2} = V(H^r) \setminus \{v_{2r}\} \supseteq V_0$. Hence $e_2 \in E_0$ — contradiction.

- $e_1 \notin E_0$ and $e_o \in E_0$
  The only two edges of E_0 which contain v_1 and do not contain v_i for odd $3 \leq i \leq 2r-1$ are e_1 and e_{i+1}. Since $e_1 \notin E_0$, the intersection of the edges of E_0 consists of the single vertex v_1 and $v_i \in e_o$ for all odd i, $3 \leq i \leq 2r-1$, it follows that $e_{i+1} \in E_0$. But then $V_0 \supseteq V(H^r) \setminus \{v_2\}$, and e_3 has to be contained in E_0 — contradiction.

- $e_1 \notin E_0$ and $e_o \notin E_0$
  In this case it has to be that $E_0 \subseteq \{e_4, e_6, \ldots, e_{2r}\}$. The only edge of e_4, e_6, …, e_{2r} which does not contain v_i for odd $i, 3 \leq i \leq 2r-1$ is e_{i+1}. Hence, it has to be that $E_0 = \{e_4, \ldots, e_{2r}\}$. But then $V_0 \supseteq V(H^r) \setminus \{v_2\}$, and thus e_3 has to be contained in E_0 — contradiction.

■

Lemma 5 *The hypergraph H^r does not contain the complete circular hypergraph C^r_{2r-1} on $2r-1$ vertices as an induced subhypergraph.*

Proof: Let $r \geq 3$ be a fixed integer throughout the proof. If H^r contains C^r_{2r-1} as an induced subhypergraph, then $H^r \setminus v_1$ is isomorphic to C^r_{2r-1} (recall that H^r is vertex-transitive). But $H^r \setminus v_1$ consists of only $2r+2-(r+1) = r+1$ edges and C^r_{2r-1} consists of $2r-1$ edges.

■

Theorem 1 now immediately follows from Lemma 2, Lemma 3, Lemma 4 and Lemma 5.

3 Conclusion

Our negative result regarding Voloshin's co-perfectness graph conjecture is not definitely a final result in the area; actually, the opposite could rather be the case. It remains a challenging problem to find all minimal non-co-perfect hypergraphs different from monostars or at least to prove whether their number is finite or not. The concept of coloring avoiding polychromatic edges is quite a new one and one may expect lots of surprising results which would show its difference (or its similarities) to the concept of usual coloring. We finish with the following two problems regarding co-perfectness of hypergraphs whose answers will be definitely of big interest:

Problem 1 *For which (r-uniform) hypergraphs H does the equality $\alpha(H) = \overline{\chi}(H)$ hold?*

Problem 2 *For which (r-uniform) hypergraphs H does the equality $\alpha(H') = \overline{\chi}(H')$ hold for all induced subhypergraph H' of H, i.e., which H are co-perfect?*

Acknowledgement

The author would like to thank Jan Kára for reading an early version of this manuscript and pointing out an error which it contained; he would also like to thank Jan Kratochvíl for introducing the concept of mixed hypergraphs and discussions on the topic. The author also appreciates comments of the anonymous referee; they helped to improve the style of the paper a lot and besides this, they contained an idea as to how to simplify the proof of Lemma 3.

References

[1] N. Alon, On a conjecture of Erdös, Simonovits, and Sós concerning anti-Ramsey theorems, *J. Graph Theory* **1** (1983), 91–94.

[2] Z. Dvořák and D. Král', On Planar Mixed Hypergraphs, *Electronic J. Combin.* **8** (1) (2001), #R35.

[3] P. Erdös, M. Simonovits and V. T. Sós, Anti-Ramsey theorems, *Coll. Math. Soc. J. Bolyai* **10**, Infinite and finite sets, Keszthely, Hungary (1973), 657–665.

[4] T. Jiang, Edge-colorings with no large polychromatic stars, to appear in *Graphs and Combinatorics*.

[5] T. Jiang, D. Mubayi, Zs. Tuza, V. Voloshin and D. B. West, The chromatic spectrum of mixed hypergraphs, *Graphs and Combinatorics* **18(2)** (2002), 309–318.

[6] T. Jiang and D. B. West, Edge-Colorings of Complete Graphs that Avoid Polychromatic Trees, to appear in *Discrete Mathematics*.

[7] T. Jiang and D. B. West, On the Erdos-Simonovits-Sos Conjecture on the anti-Ramsey number of a cycle, submitted.

[8] D. Kobler and A. Kündgen, Gaps in the chromatic spectrum of face-constrained plane graphs, *Electronic J. Combin.* **8(1)** (2001), #N3.

[9] D. Král', On Feasible Sets of Mixed Hypergraphs, submitted.,

[10] D. Král', On Maximum Face-Constrained Coloring of Plane Graphs of Girth at least 5, submitted.

[11] D. Král', J. Kratochvíl and H.-J. Voss, Mixed Hypercacti, to appear in *Discrete Mathematics*.

[12] D. Král', J. Kratochvíl and H.-J. Voss, Mixed Hypergraphs with Bounded Degree: Edge-Colouring of Mixed Multigraphs, to appear in *Theoretical Computer Science*.

[13] D. Král', J. Kratochvíl and H.-J. Voss, Mixed Hypertrees, submitted.

[14] A. Kündgen, E. Mendelsohn and V. Voloshin, Colouring planar mixed hypergraphs, *Electronic J. Combin.* **7** (2000), #R60.

[15] A. Kündgen and R. Ramamurthi, Coloring face-hypergraphs of graphs on surfaces, *J. Combin. Theory Ser. B* **85** (2002), 307–337.

[16] R. Ramamurthi and D. B. West, Maximum Face-Constrained Coloring of Plane Graphs, to appear in *Discrete Mathematics*.

[17] M. Simonovits and V. T. Sós, On restricting colorings of K_n, *Combinatorica* **4** (1984), 101–110.

[18] V. Voloshin, Coloring mixed hypergraphs: Theory, algorithms and applications, AMS, Providence, 2002.

[19] V. Voloshin: On the upper chromatic number of a hypergraph, *Australas. J. Combin.* **11** (1995), 25–45.

(Received 17/1/2002)